PGF5299: Physical Cosmology II

Problem Set 7

(Due November 04, 2014)

1) **Fisher Matrix**:

Recall the Fisher matrix components related to parameters p_α and p_β is

$$F_{\alpha\beta} = -\langle \frac{\partial^2 \ln \mathcal{L}}{\partial p_\alpha \partial p_\beta} \rangle \tag{1}$$

The covariance of the parameters can then be approximated as $C_{\alpha\beta} \approx F^{-1}_{\alpha\beta}$. Compute the Fisher matrix for the 2 cases below:

a) A Poisson Likelihood with mean/variance $\bar{\mathbf{m}} = \bar{m}_i$ for a sequence of $\mathbf{N} = N_i$ measurements:

$$\mathcal{L}(\mathbf{N}|\bar{\mathbf{m}}) = \prod_{i=1}^{b} \frac{\bar{m}_i^{N_i} e^{-\bar{m}_i}}{N_i!} \tag{2}$$

b) A Gaussian Likelihood with non-zero mean $\bar{\mathbf{m}} = \bar{m}_i$ and covariance $\mathbf{C} = C_{ij}$ for a sequence of $\mathbf{N} = N_i$ measurements:

$$\mathcal{L}(\mathbf{N}|\bar{\mathbf{m}}, \mathbf{C}) = \frac{1}{(2\pi)^{b/2} (\det \mathbf{C})^{1/2}} \exp\left[-\frac{(\mathbf{N} - \bar{\mathbf{m}})\mathbf{C}^{-1}(\mathbf{N} - \bar{\mathbf{m}})^t}{2} \right] \tag{3}$$

where

$$(\mathbf{N} - \bar{\mathbf{m}})\mathbf{C}^{-1}(\mathbf{N} - \bar{\mathbf{m}})^t = \sum_{i,j=1}^{b} (N_i - \bar{m}_i) C^{-1}_{ij} (N_j - \bar{m}_j) \tag{4}$$

Suggestion: Look at Tegmark, Taylor and Heavens, 1997.

c) Guess a form for the Fisher Matrix that interpolates between both cases above. *Suggestion:* Look at Lima and Hu, 2004.

2) MCMC:

Let us do a very simple MCMC fit. First generate a sequence of 100 pairs (x, y) where $x = 1, 2, ..., 100$ and y is drawn from a gaussian distribution $G(y) \propto \exp[(y - \bar{y})/2\sigma_y^2]$ with $\bar{y} = ax + b$ and $\sigma_y = 2$. Fix $a = 2$ and $b = 1$ to generate these 100 points. Make a plot of y versus x and show also the line $\bar{y}$ versus x superposed.

Now pretend that all you have is this sequence of *measured* 100 points (x, y). Model the likelihood of the measurements as gaussian $\mathcal{L}(y|a, b, c) = G[(y - \bar{y})^2/2\sigma_y^2]$ with variance $\sigma_y^2 = c$ and mean $\bar{y} = ax + b$. Your goal is to determine the model parameters a, b, c from the 100 measurements of y versus x.

Starting from $(a, b, c) = (0, 0, 1)$ as an initial guess point in parameter space, compute its likelihood and proceed in the usual Metropolis-Hasting steps to construct a chain for the parameters. Notice you have to choose the step size with which you'll generate new points in parameter space, so you should play with that a little bit.

Do you approach the correct solution for a, b, c? After how many steps? From the chain points, determine a 68% confidence region in the (a, b) plane after marginalizing over c (i.e. irrespective of the value of c).

Repeat the Metropolis-Hasting procedure above but start with $(a, b, c) = (1.8, 1.1, 4.2)$ as initial guesses.