PGF5299: Physical Cosmology II

Problem Set 5

(Due October 21, 2014)

1) Halo Model Ingredients

In PS4, you computed the halo mass-function for a fit from Tinker et al. 2008

$$\frac{dn(z, M_{vir})}{d\ln M_{vir}} = f(\sigma)\frac{\bar{\rho}_{m0}}{M_{vir}}\frac{d\ln\sigma^{-1}}{d\ln M_{vir}} \tag{1}$$

$$f(\sigma) = A\left[\left(\frac{\sigma}{b}\right)^{-a} + 1\right]\exp\left[-\frac{c}{\sigma^2}\right] \tag{2}$$

Now you must set values for A, a, b, c appropriate for $\Delta = \Delta_c$ (Eq. 11 below). Here $\sigma^2(M_{vir})$

$$\sigma^2(z, R) = D^2(z)\int \frac{k^2 dk}{2\pi^2}|W(kR)|^2 P_L(k) \tag{3}$$

where $M = \bar{\rho}_{m0}4\pi R^3/3$, $P_L(k)$ is the linear power spectrum at redshift $z = 0$ and

$$W(kR) = \frac{3}{k^2R^2}\left[\frac{\sin(kR)}{kR} - \cos(kR)\right] \tag{4}$$

is the Fourier Transform of a spherical top-hat window of radius R.

a) For a fiducial cosmology, compute the halo bias for the fit from Tinker et al. 2010:

$$b(z, M) = 1 - A\frac{\nu^a}{\nu^a + \delta_c^a} + B\nu^b + C\nu^c \tag{5}$$

where $\nu = \delta_c/\sigma$. Plot $b(z, M)$ versus M for $z = 0$ and $z = 1$ in the range $M = [10^{12}, 10^{16}]$. Use log scale in the M axis and linear scale for $b(M)$.

b) For a fiducial cosmology, compute the halo profile for the fit of NFW:

$$\rho(r|M_{vir}, z) = \frac{\rho_s}{cr/r_{vir}(1 + cr/r_{vir})^2} \tag{6}$$

for a halo of mass $M_{vir} = 10^{14} M_\odot/h$ at redshift $z = 0$ and plot $\rho(r)$ versus r. You will need the formulae sequence below:

$$\begin{aligned}
E^2(z) &= \Omega_m(1+z)^3 + \Omega_{DE}(1+z)^{3(1+w)}\,, & (7)\\
\rho_{crit}(z) &= \rho_{crit,0} E^2(z)\,, & (8)\\
\omega_m(z) &= \Omega_m(1+z)^3/E^2(z)\,, & (9)\\
x &= \omega_m(z) - 1\,, & (10)\\
\Delta_c &= 18\pi^2 + 82x - 39x^2\,, & (11)\\
\Delta_c &= \frac{3M_{vir}}{\rho_{crit}(z) 4\pi r_{vir}^3} \quad \rightarrow \quad \text{Find } r_{vir}\,, & (12)\\
\nu(M^*) &= 1 \quad \rightarrow \quad \text{Find } M^*\,, & (13)\\
c(M_{vir}, z) &= \frac{9}{1+z}\left[\frac{M_{vir}}{M^*}\right]^{-0.13} \quad \rightarrow \quad \text{Find } c\,, & (14)\\
M_{vir} &= 4\pi\rho_s \frac{r_{vir}^3}{c^3}\left[\ln(1+c) - \frac{c}{1+c}\right] \quad \rightarrow \quad \text{Find } \rho_s\,, & (15)
\end{aligned}$$

2) Bonus Problem: Halo Model: Combine the Halo Model ingredients you found before to compute the Halo-Model power spectrum at $z = 0$ with the formulae sequence:

$$\begin{aligned}
u(k|M) &= \int_0^{r_{vir}} dr\ 4\pi r^2 \frac{\sin kr}{kr}\frac{\rho(r|M)}{M} \qquad \text{see Cooray \& Sheth 2002} & (16)\\
P^{1h}(k) &= \int d\ln M \frac{M^2}{\bar{\rho}_{m0}^2}\frac{dn}{d\ln M}|u(k|M)|^2 & (17)\\
P^{2h}(k) &= \left[\int d\ln M \frac{M}{\bar{\rho}_{m0}}\frac{dn}{d\ln M} b(M) u(k|M)\right]^2 P_L(k) \approx P_L(k) \text{ when } k \rightarrow 0 & (18)\\
P(k) &= P^{1h}(k) + P^{2h}(k) & (19)
\end{aligned}$$

Plot $P_L(k)$, $P^{1h}(k)$, $P^{2h}(k)$ and $P(k)$, as well as the non-linear spectrum from `halofit`.