PGF5292: Physical Cosmology I

Problem Set 5

(Due May 04, 2021)

1) Distance-Redshift relation (worth 2 problems): In this problem, you will compute distances as a function of redshift numerically. Use the various comoving and physical distance definitions (radial, angular-diameter and luminosity) to plot them. For the comoving distance $D(z)$ you will need to compute numerically the integral

$$D(z) = \int_0^z \frac{dz}{H(z)} \tag{1}$$

$$H(z) = H_0\sqrt{\Omega_k(1+z)^2 + \Omega_{\rm m}(1+z)^3 + \Omega_{\rm r}(1+z)^4 + \Omega_{\rm DE}(1+z)^{3(1+w)}} \tag{2}$$

$$\Omega_k = 1 - (\Omega_{\rm m} + \Omega_{\rm r} + \Omega_{\rm DE}) \tag{3}$$

and from $D(z)$ you can compute all other distance definitions. I **highly** suggest you write a program in either Python, C/C++ or Fortran so you can easily combine with other cosmological codes later. You can then find a free numerical integrator (e.g. Simpson, Romberg, etc) to incorporate to your program.

Make a plot showing the 3 distances (radial, angular-diameter and luminosity) as a function of redshift z for the fiducial case defined in Problem Set 4. Then make plots for the same cosmology variations indicated in problem 7c) of that problem set.

2) Sandage-Loeb Test: Suppose that you measure a galaxy redshift at observation time t_o, finding $z(t_o)$. Then at time $t_o + \Delta t_o$ you measure the redshift of the same galaxy, obtaining $z(t_o + \Delta t_o)$.

a) Show that the redshift difference $\Delta z = z(t_0 + \Delta t_0) - z(t_0)$ is given by

$$\frac{\Delta z}{1+z} = \left(1 - \frac{E(z)}{1+z}\right) H_0 \Delta t_0 \tag{4}$$

where $z = z(t_0)$. For a Universe containing only matter and dark energy, we have

$$E(z) = \frac{H(z)}{H_0} = \left[\Omega_{\rm m}(1+z)^3 + \Omega_{\rm DE}(1+z)^{3(1+w)}\right]^{1/2} \tag{5}$$

b) Make a plot of $\Delta z/(1+z)$ as a function of z for a flat cosmology, i.e. $\Omega_{\rm m} + \Omega_{\rm DE} = 1$ and the combinations $(\Omega_{\rm DE}, w) = (0.5, -1); (0.7, -1); (0.9, -1); (0.7, -1.2); (0.7, -0.8)$.

c) For $\Omega_{\rm m} = 0.3$, $\Omega_\Lambda = 0.7$, and a measurement at $z = 1$, use $H_0 = 70\text{km s}^{-1}\ \text{Mpc}^{-1} = 0.72 \times 10^{-10}\ \text{year}^{-1}$ to find the fractional change in reshift if you make observations spaced in time by $\Delta t_0 = 10$ years.

Suggestion: Read the original papers: *Sandage, ApJ* **139***, 319, (1962); Loeb, ApJ.* **499***, L111 (1998).*

Note: This effect is a direct measure of the expansion. A more recent reference investigates the potential to measure this for quasars: *Corasaniti et al., Phys. Rev. D* **75***, 062001, (2007), arxiv:0701433.*

3) Dodelson 2.12

4) Dodelson 3.1