PGF5292: Physical Cosmology I

Problem Set 1

(Due March 30, 2021)

1) Lorentz Transformations: Consider a frame K′ moving with velocity v in the x direction with respect to an inertial frame K. Consider an object at rest at K′ as seen by K, as well as the fact that for a light beam $s^2 = 0$ in both frames to derive the space-time Lorentz transformations in going from K to K′. Express your answer in terms of $\beta = v/c$ and $\gamma = (1 - \beta^2)^{-1/2}$.

2) Quasar Jets: A Quasar emits relativistic blobs of plasma from near a massive black hole at its center. The blobs travels with speed v at an angle θ with respect to the line-of-sight of an observer at Earth. Because the observer can only see the blobs' movement projected onto the sky, the blobs appear to travel perpendicular to the line-of-sight with angular velocity $v_{\rm app}/r$, where r is the distance to the quasar (treating space as Euclidean, i.e. ignoring the expansion of the Universe) and $v_{\rm app}$ is the "apparent speed" of the blob.

a) Set $c = 1$, such that velocities are measured with respect to c and show that

$$v_{\rm app} = \frac{v \sin\theta}{1 - v\cos\theta}$$

b) For a given value of v, what value of θ maximizes the apparent speed? Can $v_{\rm app} > 1$?

c) Suppose $v_{\rm app} \approx 10$. What is the largest value of θ in radians?

3) Experimental Time-Dilation: On October 1971, cesium beam clocks were flown on jet flights around the world twice (eastward and westward) and then compared with reference clocks at the US Naval Observatory. From the flight paths of each trip, and considering only the special relativistic (kinematic) effect, compute how much time the clock moving eastward should have lost/gained relative to the reference clocks. Repeat the computation for the clock moving westward.

Note: On top of the kinematical effect, there is also a larger time dilation due to a gravitational effect from General Relativity. Both the kinematical and gravitational effects are comparable and necessary to explain the observed time gain/loss.

Suggestion: Read the original paper *J. Hafele, R. Keating, Science, Vol 177, No 4044 (1972), pp. 166-168*

4) Cosmic Rays and CMB: A cosmic-ray proton (mass 940 MeV) travels through space at high velocity. If the center-of-mass energy is high enough, it can collide with a cosmic microwave background (CMB) photon (whose temperature is $T = 2.74$K in its overall rest-frame) and convert into a proton plus a neutral pion (mass 140 MeV). The pion will then decay into unobservable particles, while the proton will have a lower energy than before the collision. What is the cosmic-ray energy above which we expect this process to occur, and therefore provide a cutoff in the cosmic-ray spectrum? (This is known as the Griesen-Zatsepin-Kuzmin, or GZK, cutoff.)

Note: The CMB overall rest-frame is simply the frame in which we observe no dipole anisotropy in the CMB. Obviously photons are still moving in this frame.

Suggestion: Use the relativistic energy-momentum conservation for a head-on collision in the center-of-mass such that the products will be at minimum energy, i.e. at their rest masses. Find the boost factor and Lorentz transform to get the proton's energy at the CMB rest-frame.