PGF5292: Physical Cosmology I

Problem Set 3

(Due April 02, 2019)

1) Dodelson 2.3

2) Dodelson 2.7

3) Dodelson 2.15

4) Bianchi Identities: Use the definition of the Riemann tensor in terms of the affine connection, and the definition of the affine connection in terms of the metric, to show that, in a locally inertial frame (in which $\Gamma^{\alpha}_{\mu\nu} = 0$, but not its derivatives) the covariant derivative of the Riemann Tensor is

$$R_{\lambda\mu\nu\kappa;\eta} = \frac{1}{2}\frac{\partial}{\partial x^\eta}\left(\frac{\partial^2 g_{\lambda\nu}}{\partial x^\kappa \partial x^\mu} - \frac{\partial^2 g_{\mu\nu}}{\partial x^\kappa \partial x^\lambda} - \frac{\partial^2 g_{\lambda\kappa}}{\partial x^\mu \partial x^\nu} + \frac{\partial^2 g_{\mu\kappa}}{\partial x^\nu \partial x^\lambda}\right) \tag{1}$$

Then permute indices ν, κ and η cyclically to obtain the Bianchi identities:

$$R_{\lambda\mu\nu\kappa;\eta} + R_{\lambda\mu\eta\nu;\kappa} + R_{\lambda\mu\kappa\eta;\nu} = 0 \tag{2}$$

Next, contract indices λ and ν in the above equation to obtain

$$R_{\mu\kappa;\eta} - R_{\mu\eta;\kappa} + R^{\nu}{}_{\mu\kappa\eta;\nu} = 0 \tag{3}$$

and contract indices once more to finally obtain

$$(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R)_{;\mu} = 0 \tag{4}$$

5) Einstein Equations from Action: The Einstein-Hilbert action is given by

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\; R \tag{5}$$

where R is the Ricci scalar and $g = \det g_{\mu\nu}$. Show that variation of this action with respect to the metric $g_{\mu\nu}$ and the requirement that $\delta S = 0$ leads to the Einstein Equations in vacuum, i.e.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \tag{6}$$