PGF5292: Physical Cosmology I

Problem Set 2

(Due March 26, 2019)

1) E & B Fields:

a) Write out the $\mu = 0$ component of the covariant force equation $f^\mu = qU_\nu F^{\mu\nu}$ in terms of the particle energy, velocity and the **E** & **B** fields, and provide an interpretation for it.

b) Using the Lorentz transformations on $F_{\mu\nu}$, show how **E** and **B** transform under a boost along the x-axis.

c) Show that for a general tensor $B_{\mu\nu}$, the contraction $B^{\mu\nu}B_{\mu\nu}$ is a scalar. Apply this result to the electromagnetic field tensor $F_{\mu\nu}$ to obtain the scalar in this case.

d) The energy-momentum tensor for electromagnetism is

$$T^{\mu\nu}_{(\mathrm{EM})} = F^{\mu\lambda}F^{\nu}{}_{\lambda} - \frac{1}{4}\eta^{\mu\nu}F^{\lambda\sigma}F_{\lambda\sigma}$$

Compute $T^{00}_{(\mathrm{EM})}$ and $T^{0i}_{(\mathrm{EM})}$ in terms of **E** and **B**.

2) Carroll 1.9. In addition to this problem, show that the electromagnetic energy-momentum tensor $T^{\mu\nu}_{(\mathrm{EM})}$ (see first problem above) is conserved, i.e. $\partial T^{\mu\nu}_{(\mathrm{EM})}/\partial x^\mu = 0$ when $j^\mu = 0$ (no charges nor currents).

Suggestion: Differentiate $T^{\mu\nu}_{(\mathrm{EM})}$ and use Maxwell's equations in covariant form.

3) Dodelson 2.1. In addition to this problem, express:

- The critical density today $\rho_c = 3H_0^2/8\pi G$ in units of $h^2 M_\odot \mathrm{Mpc}^{-3}$,
- c/H_0 in units of $h^{-1}\mathrm{Mpc}$.

4) Dodelson 2.2

5) Experimental Time-Dilation: Consider the same experiment with cesium clocks on jet flights around the world from Problem Set 1. Considering now only the *general relativistic* (dynamical) effect, compute how much time the clock moving eastward should have lost/gained relative to the reference clocks on Earth. Repeat for the clock moving westward.

Suggestion: Read (again!) *J. Hafele, R. Keating, Science, Vol 177, No 4044 (1972), pp. 166-168*