PGF5292: Physical Cosmology I

Problem Set 9

(Due May 13, 2015)

1) Correlation Function in Redshift Space. In class, we saw that the correlation function in redshift space is anisotropic in space and given by

$$\xi_g^s(\mathbf{s}) = \xi_g^s(s_\parallel, s_\perp) = \xi_g^s(s, \mu_s) = b^2 \sum_{l=0,2,4} c_l(\beta) L_l(\mu_s)\ \xi_l^s(s) \tag{1}$$

where $L_l(\mu_s)$ is the Legendre Polinomial of order l, $\mu_s = \cos(\theta_s)$ is the cosine of the angle between the vector $\mathbf{s}$ and the line-of-sight $\hat{\mathbf{z}}$, the coefficients

$$c_l(\beta) = \frac{2l+1}{2} \int_{-1}^{1} (1+\beta x^2)^2 L_l(x) dx = \begin{cases} 1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2, & l = 0 \\ \frac{4}{3}\beta + \frac{4}{7}\beta^2, & l = 2 \\ \frac{8}{35}\beta^2, & l = 4 \end{cases} \tag{2}$$

where $\beta = f/b$, b is the galaxy bias and $f = \frac{d\ln D}{d\ln a}$, and the multipoles

$$\xi_l^s(s) = i^l \int \frac{k^2 dk}{2\pi^2} j_l(ks) P^r(k) \tag{3}$$

a) From the real-space matter power spectrum $P^r(k)$ (e.g. from CAMB), use Eq.(3) to compute the multipoles $\xi_l^s(s)$ for $l = 0, 2, 4$. Plot them in log-scale and apropriate ranges.

b) Assuming $b = 1$, use Eq.(2) to compute the coefficients $c_l(\beta)$. If you have a numerical growth function D, use it to compute f. Otherwise use a fitting function (e.g. $f = \Omega_m^\gamma(z)$).

c) Finally, use Eq.(1) to obtain the correlation function as a function of parallel and perpendicular directions at $z = 0$ and 1.0 and make a 2D plot of your results, with $s_\perp = s\sin\theta$ in the x-axis and $s_\parallel = s\cos\theta_s$ in the y-axis, and a color code for the value of $\xi^s(s_\perp, s_\parallel)$. Compare these results to a similar 2D plot for the isotropic real space correlation $\xi^r(s)$.

2) Multipoles and Bessel Function. Use Eq.(3) and the properties of spherical Bessell functions $j_l(x)$ to show that the multipoles can also be written in terms of the real-space correlation function $\xi^r(x)$ as

$$
\begin{aligned}
\xi_0^s(s) &= \int \frac{k^2 dk}{2\pi^2} \frac{\sin(ks)}{ks} P^r(k) = \xi^r(s) \\
\xi_2^s(s) &= \left(\frac{3}{s^3} \int_0^s dx \; x^2 \xi^r(x) \right) - \xi^r(s) \\
\xi_4^s(s) &= \frac{5}{2} \left(\frac{3}{s^3} \int_0^s dx \; x^2 \xi^r(x) \right) - \frac{7}{2} \left(\frac{5}{s^5} \int_0^s dx \; x^4 \xi^r(x) \right) + \xi^r(s)
\end{aligned}
$$

This is useful to propagate into redshift space, non-linear prescriptions that modify the real-space power spectrum or correlation, i.e. from a prescription $\xi^r(x) \to \xi^r_{\mathrm{NL}}(x)$ one can obtain further $\xi^s_{\mathrm{NL}}(\mathbf{s})$. Notice that

$$
\bar{\xi}(s) = \frac{3}{s^3} \int_0^s dx \; x^2 \xi^r(x) = \frac{3}{4\pi s^3} \int_0^s d^3x \; \xi^r(x)
$$

represents the *volume average* of ξ up to radius s.