# PGF5292: Physical Cosmology I

## Problem Set 3

(Due April 01, 2015)

**1) Scale Factor Evolution**: In this problem, you will solve the Friedmann equation **numerically**.

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\underbrace{\Omega_k a^{-2}}_{\text{Curvature}} + \underbrace{\Omega_{\rm m} a^{-3}}_{\text{Matter}} + \underbrace{\Omega_{\rm r} a^{-4}}_{\text{Radiation}} + \underbrace{\Omega_{\rm DE} a^{-3(1+w)}}_{\text{Dark Energy}}\right] \quad (1)$$

$$\text{where} \quad \Omega_k = 1 - (\Omega_{\rm m} + \Omega_{\rm r} - \Omega_{\rm DE}) \quad (2)$$

For that, you must write a numerical program that uses a differential equation solver (e.g. Runge-Kutta or better). I **highly** suggest you use something like C/C++, Fortran or Python, since it will make it easier to integrate with other cosmological codes in the future. If you don't know any of these languages, it is a good time to learn. You can then try to find free efficient numerical solvers for differential equations to include into your program.

In each case below, set up appropriate **initial conditions** using the *dominant* component at early times. For instance, for a universe with matter and radiation, at early times radiation dominates and you may find the analytical solutions $a(t)$ to set the correct value of $a_0 = a(t_0)$ at the initial time $t_0$.

Note that the only quantity with units here is $H_0$ (units of time$^{-1}$). Use $H_0$ such that you present your results with time in Gigayears (Gyr). For all cases, fix $h = 0.72$.

a) First do the **single-component** cases. Leave one $\Omega_i$ at a time, and set all others equal to zero. For the dark energy, choose a cosmological constant, i.e. $w = -1$. For each case, plot the numerically derived scale factor as a function of time and compare your numerical solution to the analytical solution, plotting also the analytical solution.

b) Now do an intermediate **two-component** case, containing matter + cosmological constant. In this case you can also find an analytical solution to compare.

c) Now do the **complete** case with all terms. Use **fiducial** values: $\Omega_k = 0$ (flat universe), $\Omega_{\rm m} = 0.25$, $\Omega_{\rm DE} = 0.75$, $w = -1$, $\Omega_{\rm r} = 8.2 \times 10^{-5}$. Compute the **age of the Universe**, by finding the time $t_0$ that corresponds to today, i.e. the time when $a(t_0) = 1$. Change the parameter values one at a time ($\Omega$'s and $w$ for cases below) and **plot** the corresponding $a(t)$ for each variation set. See also the impact on the age of the Universe.

i) $\Omega_{\rm m} = 0.1, 0.25, 0.5, 1.0$ (with all other parameters equal to fiducial)

ii) $\Omega_{\rm DE} = 0.5, 0.75, 1.0$ (with all other parameters equal to fiducial)

iii) $w = -0.8, -1.0, -1.2$ (with all other parameters equal to fiducial)

iv) $\Omega_{\rm r} = (6, 8, 10) \times 10^{-5}$ (with all other parameters equal to fiducial)

v) Flat cases: $(\Omega_{\rm m}, \Omega_{\rm DE}) = (0.0, 1.0), (0.1, 0.9), (0.25, 0.75), (0.75, 0.25), (1.0, 0.0)$.

Indicate clearly in the plots, the cases you are showing.

## 2) **Distance-Redshift relation**: In this problem, you will compute distances as a function of redshift **numerically**. For the comoving radial distance $D(z)$ you will need to compute numerically the integral

$$
\begin{aligned}
D(z) &= \int_0^z \frac{dz}{H(z)} && (3)\\
H(z) &= H_0\sqrt{\Omega_k(1+z)^2 + \Omega_{\rm m}(1+z)^3 + \Omega_{\rm r}(1+z)^4 + \Omega_{\rm DE}(1+z)^{3(1+w)}} && (4)\\
\Omega_k &= 1 - (\Omega_{\rm m} + \Omega_{\rm r} + \Omega_{\rm DE}) && (5)
\end{aligned}
$$

From $D(z)$ you can obtain other distance definitions. I **highly** suggest you write a program in C/C++ or Fortran so you can easily combine with other cosmological codes later. You can then find a free numerical integrator (e.g. Simpson, Romberg, etc) to incorporate to your program. Plot the 3 distances (radial, angular-diameter and luminosity) as a function of redshift $z$ for the fiducial case and cosmology variations indicated in problem 1).