PGF5292: Physical Cosmology I

Problem Set 2

(Due March 25, 2015)

1) Experimental Time-Dilation: Consider the same experiment with cesium clocks on jet flights around the world from Problem Set 1. Considering now only the *general relativistic* (dynamical) effect, compute how much time the clock moving eastward should have lost/gained relative to the reference clocks on Earth. Repeat for the clock moving westward.

Suggestion: Read (again!) *J. Hafele, R. Keating, Science, Vol 177, No 4044 (1972), pp. 166-168*

2) Dodelson 2.3

3) Dodelson 2.15

4) Bianchi Identities: Use the definition of the Riemann tensor in terms of the affine connection, and the definition of the affine connection in terms of the metric, to show that, in a locally inertial frame (in which $\Gamma^{\alpha}_{\mu\nu} = 0$, but not its derivatives) the covariant derivative of the Riemann Tensor is

$$R_{\lambda\mu\nu\kappa;\eta} = \frac{1}{2}\frac{\partial}{\partial x^{\eta}}\left(\frac{\partial^2 g_{\lambda\nu}}{\partial x^{\kappa}\partial x^{\mu}} - \frac{\partial^2 g_{\mu\nu}}{\partial x^{\kappa}\partial x^{\lambda}} - \frac{\partial^2 g_{\lambda\kappa}}{\partial x^{\mu}\partial x^{\nu}} + \frac{\partial^2 g_{\mu\kappa}}{\partial x^{\nu}\partial x^{\lambda}}\right) \tag{1}$$

Then permute indices ν, κ and η cyclically to obtain the Bianchi identities:

$$R_{\lambda\mu\nu\kappa;\eta} + R_{\lambda\mu\eta\nu;\kappa} + R_{\lambda\mu\kappa\eta;\nu} = 0 \tag{2}$$

Next, contract indices λ and ν in the above equation to obtain

$$R_{\mu\kappa;\eta} - R_{\mu\eta;\kappa} + R^{\nu}{}_{\mu\kappa\eta;\nu} = 0 \tag{3}$$

and contract indices once more to finally obtain

$$(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R)_{;\mu} = 0 \tag{4}$$

5) Einstein Equations from Action: The Einstein-Hilbert action is given by

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\ R \tag{5}$$

where R is the Ricci scalar and $g = \det g_{\mu\nu}$. Show that variation of this action with respect to the metric $g_{\mu\nu}$ and the requirement that $\delta S = 0$ leads to the Einstein Equations in vacuum, i.e.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0 \tag{6}$$