

# PGF5292: Physical Cosmology I

## Problem Set 10

(Due May 20, 2015)

### 1) Galaxy Cluster Abundance

Last semester you computed the variance  $\sigma^2$  of linear fluctuations on a scale  $R$

$$\sigma^2(z, R) = D^2(z) \int \frac{k^2 dk}{2\pi^2} |W(kR)|^2 P_L(k) = D^2(z) \sigma^2(z=0, R) \quad (1)$$

where  $P_L(k)$  is the linear matter power spectrum at redshift  $z=0$  (e.g. from CAMB) and

$$W(kR) = \frac{3}{k^2 R^2} \left[ \frac{\sin(kR)}{kR} - \cos(kR) \right] \quad (2)$$

is the Fourier Transform of a spherical top-hat window of radius  $R$ .

a) Use your previous results to compute  $\sigma(z, M) = D(z)\sigma(z=0, M)$  at a scale  $R$  that encloses mass  $M$  at the background density  $\bar{\rho}_{m0}$  today. You just need to convert from radius to mass using  $M = \bar{\rho}_{m0} 4\pi R^3/3$ . Plot  $\sigma(z, M)$  versus  $M$  for  $z=0$  and  $z=1$  in log scale, for the range  $M = [10^{12}, 10^{16}]$  and choose an appropriate range in the y-axis. What value of  $M$  corresponds to  $\sigma(z=0, M) = \delta_c = 1.686$  ?

It will be useful for the next items if you compute  $\sigma(z=0, M)$  for certain values of  $M$  and define an interpolating function (e.g. spline) that gives you  $\sigma(z=0, M)$  for any values of  $M$  (check that you have a sufficient number of points for the interpolation to work well). Then  $\sigma(z, M) = D(z)\sigma(z=0, M)$  gives you  $\sigma$  for any  $z$  and  $M$ .

b) Compute  $d\sigma/dM$  by finite difference of the previous result, and use this to compute

$$\frac{d \ln \sigma^{-1}}{d \ln M} = -\frac{M}{\sigma} \frac{d\sigma}{dM} \quad (3)$$

Plot  $d \ln \sigma^{-1}/d \ln M$  versus  $M$  in the same mass range as in a). Again, define an interpolating function that gives you this function at any value of  $M$ .

c) Use the results from a) and b) to compute the halo mass function as

$$\frac{dn(z, M)}{d \ln M} = f(\sigma) \frac{\bar{\rho}_m}{M} \frac{d \ln \sigma^{-1}}{d \ln M} \quad (4)$$

for the fit from Tinker et al. 2008, i.e.

$$f(\sigma) = A \left[ \left( \frac{\sigma}{b} \right)^{-a} + 1 \right] \exp \left[ -\frac{c}{\sigma^2} \right] \quad (5)$$

and choose values for  $A, a, b, c$  that are appropriate for  $\Delta = 200$  (see Tinker's paper). Plot  $dn/d \ln M$  versus  $M$  in the same range as in a), for  $z = 0$  and  $z = 1$ .

d) Integrate  $dn/d \ln M$  in mass  $M$  for masses above  $M_{\text{lim}} = 10^{14} M_{\odot}/h$  for various values of  $z$  and interpolate to finally obtain the number density  $n(z)$  at any  $z$ :

$$n(z) = \int_{M_{\text{lim}}}^{\infty} d \ln M \frac{dn(z, M)}{d \ln M} \quad (6)$$

Plot  $n(z)$  versus  $z$ , for  $z = [0, 2]$ .

e) Finally, integrate  $n(z)$  in comoving volume  $dV = \Delta \Omega dz D_A^2(z)/H(z)$ , for  $\Delta \Omega = 5000 \text{ deg}^2$  (convert  $\text{deg}^2 \rightarrow \text{rad}^2$ ) to find the number  $N(z_i)$  of halos in redshift bins of width  $\Delta z = 0.1$ :

$$N(z_i) = \Delta \Omega \int_{z_i}^{z_i + \Delta z} dz \frac{D_A^2(z)}{H(z)} n(z) \quad (7)$$

Plot  $N(z_i)$  versus  $z_i$  for 20 bins in  $z_i$ , i.e. from 0 to 2.