PGF5292: Physical Cosmology I

Problem Set 1

(Due March 18, 2015)

1) Quasar Jets: A Quasar emits relativistic blobs of plasma from near a massive black hole at its center. The blobs travels with speed v at an angle θ with respect to the line-of-sight of an observer at Earth. Because the observer can only see the blobs' movement projected onto the sky, the blobs appear to travel perpendicular to the line-of-sight with angular velocity v_{app}/r, where r is the distance to the quasar (treating space as Euclidean, i.e. ignoring the expansion of the Universe) and v_{app} is the "apparent speed"of the blob.

a) Set $c = 1$, such that velocities are measured with respect to c and show that

$$v_{\text{app}} = \frac{v \sin\theta}{1 - v\cos\theta}$$

b) For a given value of v, what value of θ maximizes the apparent speed? Can $v_{\text{app}} > 1$?

c) Suppose $v_{\text{app}} \approx 10$. What is the largest value of θ in radians?

2) Experimental Time-Dilation: On October 1971, cesium beam clocks were flown on jet flights around the world twice (eastward and westward) and then compared with reference clocks at the US Naval Observatory. From the flight paths of each trip, and considering only the special relativistic (kinematic) effect, compute how much time the clock moving eastward should have lost/gained relative to the reference clocks. Repeat the computation for the clock moving westward.

Note: On top of the kinematical effect, there is also a larger time dilation due to a gravitational effect from General Relativity. Both the kinematical and gravitational effects are comparable and necessary to explain the observed time gain/loss.

Suggestion: Read the original paper *J. Hafele, R. Keating, Science, Vol 177, No 4044 (1972), pp. 166-168*

3) **E & B Fields**:

a) Write out the $\mu = 0$ component of the covariant force equation $f^\mu = qU_\nu F^{\mu\nu}$ in terms of the particle energy, velocity and the **E** & **B** fields, and provide an interpretation for it.

b) Using the Lorentz transformations on $F_{\mu\nu}$, show how **E** and **B** transform under a boost along the x-axis.

c) Show that for a general tensor $B_{\mu\nu}$, the contraction $B^{\mu\nu}B_{\mu\nu}$ is a scalar. Apply this result to the electromagnetic field tensor $F_{\mu\nu}$ to obtain the scalar in this case.

d) The energy-momentum tensor for electromagnetism is

$$T^{\mu\nu}_{(\mathrm{EM})} = F^{\mu\lambda}F^{\nu}{}_{\lambda} - \frac{1}{4}\eta^{\mu\nu}F^{\lambda\sigma}F_{\lambda\sigma}$$

Compute $T^{00}_{(\mathrm{EM})}$ and $T^{0i}_{(\mathrm{EM})}$ in terms of **E** and **B**.

4) **Carroll 1.9**. In addition to this problem, show that the electromagnetic energy-momentum tensor $T^{\mu\nu}_{(\mathrm{EM})}$ (see third problem above) is conserved, i.e. $\partial T^{\mu\nu}_{(\mathrm{EM})}/\partial x^\mu = 0$ when $j^\mu = 0$ (no charges nor currents).

Suggestion: Differentiate $T^{\mu\nu}_{(\mathrm{EM})}$ and use Maxwell's equations in covariant form.

5) **Dodelson 2.1**. In addition to this problem, express:

- The critical density today $\rho_c = 3H_0^2/8\pi G$ in units of $h^2 M_\odot \mathrm{Mpc}^{-3}$,
- c/H_0 in units of $h^{-1}\mathrm{Mpc}$.