Apêndice B

Derivada de $\log\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}-a}\right)$

Vamos usar

$$\begin{aligned}
\log(x/y) &= \log(x) - \log(y) && \text{(B.1)}\\
\frac{d}{dx}\log(x) &= 1/x && \text{(B.2)}\\
\frac{d}{dx}\sqrt{x} &= \frac{1}{2\sqrt{x}} && \text{(B.3)}\\
\frac{d}{dx}x^2 &= 2x. && \text{(B.4)}
\end{aligned}$$

Para

$$V = \log\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}-a}\right) \tag{B.5}$$

Temos

$$V = \log\left(\sqrt{x^2+a^2}+a\right) - \log\left(\sqrt{x^2+a^2}-a\right) \tag{B.6}$$

e portanto

$$\begin{aligned}
\frac{dV}{dx} &= \frac{x}{\sqrt{x^2+a^2}(\sqrt{x^2+a^2}+a)} - \frac{x}{\sqrt{x^2+a^2}(\sqrt{x^2+a^2}-a)}\\
&= \frac{x}{\sqrt{x^2+a^2}}\left(\frac{1}{\sqrt{x^2+a^2}+a} - \frac{1}{\sqrt{x^2+a^2}-a}\right)\\
&= \frac{x}{\sqrt{x^2+a^2}}\left(\frac{\sqrt{x^2+a^2}-a-\sqrt{x^2+a^2}-a}{(x^2+a^2)-a^2}\right)\\
&= \frac{x}{\sqrt{x^2+a^2}}\left(\frac{-2a}{x^2}\right)\\
&= -\frac{2a}{x\sqrt{x^2+a^2}} && \text{(B.7)}
\end{aligned}$$