

## Apêndice D

### Rotacional de $\vec{V}[f(r)]$

Suponha um vetor  $\vec{V}$  que dependa apenas de uma função  $f(r)$ , que por sua vez depende apenas da magnitude do vetor radial  $\vec{r} = (x, y, z)$ . Da definição de rotacional, temos:

$$\begin{aligned}\vec{\nabla} \times \vec{V}[f(r)] &= \left( \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}, \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}, \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) \\ &= \left( \frac{dV_z}{df} \frac{df}{dr} \frac{\partial r}{\partial y} - \frac{dV_y}{df} \frac{df}{dr} \frac{\partial r}{\partial z}, \frac{dV_x}{df} \frac{df}{dr} \frac{\partial r}{\partial z} - \frac{dV_z}{df} \frac{df}{dr} \frac{\partial r}{\partial x}, \frac{dV_y}{df} \frac{df}{dr} \frac{\partial r}{\partial x} - \frac{dV_x}{df} \frac{df}{dr} \frac{\partial r}{\partial y} \right)\end{aligned}$$

Como  $\partial r / \partial x = x/r$ , etc, temos

$$\begin{aligned}\vec{\nabla} \times \vec{V}[f(r)] &= \frac{df}{dr} \frac{1}{r} \left( y \frac{dV_z}{df} - z \frac{dV_y}{df}, z \frac{dV_x}{df} - x \frac{dV_z}{df}, x \frac{dV_y}{df} - y \frac{dV_x}{df} \right) \\ &= \frac{df}{dr} \frac{1}{r} \underbrace{(x, y, z)}_{\vec{r}} \times \underbrace{\left( \frac{dV_x}{df}, \frac{dV_y}{df}, \frac{dV_z}{df} \right)}_{\frac{d\vec{V}}{df}} \\ &= \frac{df}{dr} \frac{1}{r} \left( \vec{r} \times \frac{d\vec{V}}{df} \right) \tag{D.1}\end{aligned}$$

$$= \frac{df}{dr} \hat{r} \times \frac{d\vec{V}}{df} \tag{D.2}$$

Para  $f(r)$ , temos  $\vec{\nabla} f = \frac{df}{dr} \hat{r}$ , portanto:

$$\vec{\nabla} \times \vec{V}[f(r)] = \vec{\nabla} f(r) \times \frac{d\vec{V}}{df} = -\frac{d\vec{V}}{df} \times \vec{\nabla} f(r) \tag{D.3}$$