

Apêndice C

Gradiente de $f(r)$

Suponha uma função f que dependa apenas da magnitude do vetor radial $\vec{r} = (x, y, z)$, ou seja da coordenada radial r em coordenadas esféricas, i.e. $f = f(r)$. Temos que

$$r = \sqrt{x^2 + y^2 + z^2} \quad (\text{C.1})$$

O gradiente de f em coordenadas cartesianas é

$$\vec{\nabla}f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right) \quad (\text{C.2})$$

Temos por exemplo que

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r} \frac{\partial r}{\partial x} = \frac{\partial f}{\partial r} \frac{2x}{2\sqrt{x^2 + y^2 + z^2}} = \frac{x}{r} \frac{\partial f}{\partial r} \quad (\text{C.3})$$

Similarmente,

$$\frac{\partial f}{\partial y} = \frac{y}{r} \frac{\partial f}{\partial r} \quad (\text{C.4})$$

$$\frac{\partial f}{\partial z} = \frac{z}{r} \frac{\partial f}{\partial r} \quad (\text{C.5})$$

Assim,

$$\vec{\nabla}f = \left(\frac{x}{r} \frac{\partial f}{\partial r}, \frac{y}{r} \frac{\partial f}{\partial r}, \frac{z}{r} \frac{\partial f}{\partial r} \right) = \frac{1}{r} \frac{\partial f}{\partial r} (x, y, z) = \frac{\partial f}{\partial r} \frac{\vec{r}}{r} \quad (\text{C.6})$$

Portanto, como $\hat{r} = \vec{r}/r$:

$$\vec{\nabla}f(r) = \frac{\partial f}{\partial r} \hat{r} \quad (\text{C.7})$$