Apêndice D

Rotacional de $\vec{V}[f(r)]$

Suponha um vetor $\vec{V}$ que dependa apenas de uma função $f(r)$, que por sua vez depende apenas da magnitude do vetor radial $\vec{r} = (x, y, z)$. Da definição de rotacional, temos:

$$\begin{aligned}
\vec{\nabla} \times \vec{V}[f(r)] &= \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}, \frac{\partial V_x}{\partial z} - \frac{\partial V_x}{\partial x}, \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) \\
&= \left(\frac{dV_z}{df}\frac{df}{dr}\frac{\partial r}{\partial y} - \frac{dV_y}{df}\frac{df}{dr}\frac{\partial r}{\partial z}, \frac{dV_x}{df}\frac{df}{dr}\frac{\partial r}{\partial z} - \frac{dV_x}{df}\frac{df}{dr}\frac{\partial r}{\partial x}, \frac{dV_y}{df}\frac{df}{dr}\frac{\partial r}{\partial x} - \frac{dV_x}{df}\frac{df}{dr}\frac{\partial r}{\partial y} \right)
\end{aligned}$$

Como $\partial r/\partial x = x/r$, etc, temos

$$\begin{aligned}
\vec{\nabla} \times \vec{F}[f(r)] &= \frac{df}{dr}\frac{1}{r}\left(y\frac{dV_z}{df} - z\frac{dV_y}{df}, z\frac{dV_x}{df} - x\frac{dV_x}{df}, x\frac{dV_y}{df} - y\frac{dV_x}{df} \right) \\
&= \frac{df}{dr}\frac{1}{r}\underbrace{(x, y, z)}_{\vec{r}} \times \underbrace{\left(\frac{dV_x}{df}, \frac{dV_y}{df}, \frac{dV_z}{df} \right)}_{\frac{d\vec{V}}{df}} \\
&= \frac{df}{dr}\frac{1}{r}\left(\vec{r} \times \frac{d\vec{V}}{df} \right) \qquad \text{(D.1)} \\
&= \frac{df}{dr}\hat{r} \times \frac{d\vec{V}}{df} \qquad \text{(D.2)}
\end{aligned}$$

Para $f(r)$, temos $\vec{\nabla} f = \frac{df}{dr}\hat{r}$, portanto:

$$\vec{\nabla} \times \vec{V}[f(r)] = \vec{\nabla} f(r) \times \frac{d\vec{V}}{df} = -\frac{d\vec{V}}{df} \times \vec{\nabla} f(r) \qquad \text{(D.3)}$$