Apêndice B

Derivada de $\log\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}-a}\right)$

Vamos usar

$$\log(x/y) = \log(x) - \log(y) \tag{B.1}$$

$$\frac{d}{dx}\log(x) = 1/x \tag{B.2}$$

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}} \tag{B.3}$$

$$\frac{d}{dx}x^2 = 2x. \tag{B.4}$$

Para

$$V = \log\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}-a}\right) \tag{B.5}$$

Temos

$$V = \log\left(\sqrt{x^2+a^2}+a\right) - \log\left(\sqrt{x^2+a^2}-a\right) \tag{B.6}$$

e portanto

$$\begin{aligned}
\frac{dV}{dx} &= \frac{x}{\sqrt{x^2+a^2}(\sqrt{x^2+a^2}+a)} - \frac{x}{\sqrt{x^2+a^2}(\sqrt{x^2+a^2}-a)} \\
&= \frac{x}{\sqrt{x^2+a^2}}\left(\frac{1}{\sqrt{x^2+a^2}+a} - \frac{1}{\sqrt{x^2+a^2}-a}\right) \\
&= \frac{x}{\sqrt{x^2+a^2}}\left(\frac{\sqrt{x^2+a^2}-a-\sqrt{x^2+a^2}-a}{(x^2+a^2)-a^2}\right) \\
&= \frac{x}{\sqrt{x^2+a^2}}\left(\frac{-2a}{x^2}\right) \\
&= -\frac{2a}{x\sqrt{x^2+a^2}}
\end{aligned} \tag{B.7}$$