4300430: Introdução à Cosmologia Física

Problem Set 6

(Due October 17, 2024)

1) CAMB: (worth 6 points)

Download and install CAMB at `camb.info`. This is a Fortran code which evolves the linear perturbations for the Einstein-Boltzmann equations accounting for various species (dark matter, baryons, photons, neutrinos, scalar fields, etc.). Notice CAMB can also be used within Python. Run the code to generate the matter power spectrum $P(k)$ today, i.e. at $z = 0$.

Plot $P(k)$ versus k in logarithmic scale (indicating units in the axis!), for the values of cosmological parameters from problem 1 in Problem Set 3. Describe (using the plots) the effects of changing the various parameters.

2) Top-hat Window (worth 1 point)

Define the radial top-hat window function $W(\mathbf{x}, R)$ on a scale R as

$$W(\mathbf{x}, R) = \begin{cases} 3/(4\pi R^3) & \text{if } r < R\,, \\ 0 & \text{if } r > R\,, \end{cases} \tag{1}$$

where $r = |\mathbf{x}|$. Show that the Fourier Transform of $W(\mathbf{x}, R)$ is given by

$$\tilde{W}(kR) = \frac{3}{k^2R^2}\left[\frac{\sin(kR)}{kR} - \cos(kR)\right] \tag{2}$$

3) Filtered Variance (worth 3 points)

Use the CAMB power spectrum $P(k)$ for the fiducial cosmology in problem 1) to com-

pute (numerically) the variance of the linear density field filtered on a scale R:

$$\sigma_R^2 \;=\; \int \frac{d^3k}{(2\pi)^3}\, P(k)|\tilde{W}(kR)|^2 = \frac{1}{2\pi^2}\int_0^\infty dk\; k^2 P(k)|\tilde{W}(kR)|^2\,, \tag{3}$$

where $\tilde{W}(kR)$ is given in problem 2). Obviously the numerical integral cannot be done in the formal limits above, so make some convergence test to assure your answer does not depend on the specific range used.

Plot σ_R versus R in logarithmic scale from $R = 10^{-2}\ h^{-1}$Mpc to $R = 10^2\ h^{-1}$Mpc.

What is the value you obtain for σ_R when $R = 8\ h^{-1}$Mpc? This is known as σ_8 and is typically quoted as a useful cosmological parameter related to the amplitude of the spectrum at $z = 0$. How does the value you obtain compare to the value outputted by CAMB? How does it compare to the value obtained by recent experiments (e.g. from the Planck satellite)?