4300430: Introdução à Cosmologia Física

Problem Set 3

(Due September 24, 2024)

1) **Scale Factor Evolution**: (worth 10.0 points)

In this problem, you will solve the Friedmann equation **numerically**.

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\underbrace{\Omega_k a^{-2}}_{\text{Curvature}} + \underbrace{\Omega_{\rm m} a^{-3}}_{\text{Matter}} + \underbrace{\Omega_{\rm r} a^{-4}}_{\text{Radiation}} + \underbrace{\Omega_{\rm DE} a^{-3(1+w)}}_{\text{Dark Energy}}\right] \tag{1}$$

$$\text{where} \quad \Omega_k = 1 - (\Omega_{\rm m} + \Omega_{\rm r} + \Omega_{\rm DE}) \tag{2}$$

For that, you must write a numerical program that uses a differential equation solver (e.g. Runge-Kutta or better). I **highly** suggest you use something like C/C++, Fortran or Python, since it will make it easier to integrate with other cosmological codes in the future. If you don't know any of these languages, it is a good time to learn. You can then try to find free efficient numerical solvers for differential equations to include into your program.

In each case below, set up appropriate **initial conditions** using the *dominant* component at early times. For instance, for a universe with matter and radiation, at early times radiation dominates and you may use the analytical solution for $a(t)$ to set the correct value of $a_0 = a(t_0)$ at the initial time t_0.

Note that the only quantity with units here is H_0 (units of time^{-1}). Use H_0 such that you present your results with time in Gigayears (Gyr). For all cases, fix $h = 0.72$.

a) (worth 2 points) First do the **single-component** cases. Leave one $\Omega_i = 1$ at a time, and set all others equal to zero (notice the constraint in Eq. (2) above). For the dark energy, choose a cosmological constant, i.e. $w = -1$. For each case, plot the numerically derived scale factor as a function of time and compare your numerical solution to the analytical solution, plotting also the analytical solution.

b) (worth 3 points) Now do intermediate **two-component** cases, containing i) matter + cosmological constant and ii) matter + curvature (choose closed universes ($k > 0$), i.e. $\Omega_k < 0$ and $\Omega_{\rm m} > 1.0$). In these cases you also have analytical solutions to compare.

c) (worth 5 points) Now do the **complete** case with all terms. Use **fiducial** values: $\Omega_k = 0$ (flat universe), $\Omega_{\rm m} = 0.25$, $\Omega_{\rm DE} = 0.75$, $w = -1$, $\Omega_{\rm r} = 8.2 \times 10^{-5}$. Compute the **age of the Universe**, by finding the time t_0 that corresponds to today, i.e. the time when $a(t_0) = 1$. Change the parameter values one at a time (Ω's and w for cases below) and **plot** the corresponding $a(t)$ for each variation set. See also the impact on the age of the Universe.

i) $\Omega_{\rm m} = 0.1, 0.25, 0.5, 1.0$ (with all other parameters equal to fiducial)

ii) $\Omega_{\rm DE} = 0.5, 0.75, 1.0$ (with all other parameters equal to fiducial)

iii) $w = -0.8, -1.0, -1.2$ (with all other parameters equal to fiducial)

iv) $\Omega_{\rm r} = (6, 8, 10) \times 10^{-5}$ (with all other parameters equal to fiducial)

v) Flat cases: $(\Omega_{\rm m}, \Omega_{\rm DE}) = (0.0, 1.0), (0.1, 0.9), (0.25, 0.75), (0.75, 0.25), (1.0, 0.0)$.

Indicate clearly in the plots the cases you are showing.

ps: Notice that you must always impose the constraint in Eq. (2), so it is not really possible to keep ALL other parameters equal to fiducial when you change one of the fiducial values. You may choose to change Ω_k appropriately as you change each parameter, except for the flat cases v), where by construction $\Omega_k = 0$ always.