4300430: Introdução à Cosmologia Física

Problem Set 2

(Due September 10, 2024)

1) Experimental Time-Dilation: (worth 2.5 points)

Consider the same experiment with cesium clocks on jet flights around the world from Problem Set 1. Considering now only the *general relativistic* (dynamical) effect, compute how much time the clock moving eastward should have lost/gained relative to the reference clocks on Earth. Repeat for the clock moving westward.

Suggestion: Read (again!) *J. Hafele, R. Keating, Science, Vol 177, No 4044 (1972), pp. 166-168*

2) Dodelson 2.3 (worth 2.5 points)

3) Dodelson 2.8 (worth 2.5 points)

4) Bianchi Identities: (worth 2.5 points)

Use the definition of the Riemann tensor in terms of the affine connection, and the definition of the affine connection in terms of the metric, to show that, in a locally inertial frame (in which $\Gamma^{\alpha}_{\mu\nu} = 0$, but not its derivatives) the covariant derivative of the Riemann Tensor is

$$R_{\lambda\mu\nu\kappa;\eta} = \frac{1}{2}\frac{\partial}{\partial x^{\eta}}\left(\frac{\partial^2 g_{\lambda\nu}}{\partial x^{\kappa}\partial x^{\mu}} - \frac{\partial^2 g_{\mu\nu}}{\partial x^{\kappa}\partial x^{\lambda}} - \frac{\partial^2 g_{\lambda\kappa}}{\partial x^{\mu}\partial x^{\nu}} + \frac{\partial^2 g_{\mu\kappa}}{\partial x^{\nu}\partial x^{\lambda}}\right) \tag{1}$$

Then permute indices ν, κ and η cyclically to obtain the Bianchi identities:

$$R_{\lambda\mu\nu\kappa;\eta} + R_{\lambda\mu\eta\nu;\kappa} + R_{\lambda\mu\kappa\eta;\nu} = 0 \tag{2}$$

Next, contract indices λ and ν in the above equation to obtain

$$R_{\mu\kappa;\eta} - R_{\mu\eta;\kappa} + R^{\nu}{}_{\mu\kappa\eta;\nu} = 0 \tag{3}$$

and contract indices once more to finally obtain

$$(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R)_{;\mu} = 0 \tag{4}$$