

4300430: Introdução à Cosmologia Física

Problem Set 10

(Due November 14, 2024)

1) Galaxy Cluster Abundance

In PS 6 you computed the variance σ^2 of linear fluctuations on a scale R

$$\sigma^2(z, R) = D^2(z) \int \frac{k^2 dk}{2\pi^2} |W(kR)|^2 P_L(k) = D^2(z) \sigma^2(z=0, R) \quad (1)$$

where $P_L(k)$ is the linear matter power spectrum at redshift $z=0$ (e.g. from CAMB) and

$$W(kR) = \frac{3}{k^2 R^2} \left[\frac{\sin(kR)}{kR} - \cos(kR) \right] \quad (2)$$

is the Fourier Transform of a spherical top-hat window of radius R .

a) Use your previous results to compute $\sigma(z, M) = D(z)\sigma(z=0, M)$ at a scale R that encloses mass M at the background density $\bar{\rho}_{m0}$ today. You just need to convert from radius to mass using $M = \bar{\rho}_{m0} 4\pi R^3/3$. Plot $\sigma(z, M)$ versus M for $z=0$ and $z=1$ in log scale, for the range $M = [10^{12}, 10^{16}]M_\odot/h$ and choose an appropriate range in the y-axis. What value of M corresponds to $\sigma(z=0, M) = \delta_c = 1.686$?

It will be useful for the next items if you compute $\sigma(z=0, M)$ for certain values of M and define an interpolating function (e.g. spline) that gives you $\sigma(z=0, M)$ for any values of M (check that you have a sufficient number of points for the interpolation to work well). Then $\sigma(z, M) = D(z)\sigma(z=0, M)$ gives you σ for any z and M .

b) Compute $d\sigma/dM$ by finite difference of the previous result, and use this to compute

$$\frac{d \ln \sigma^{-1}}{d \ln M} = -\frac{M}{\sigma} \frac{d\sigma}{dM} \quad (3)$$

Plot $d \ln \sigma^{-1}/d \ln M$ versus M in the same mass range as in a). Again, define an interpolating function that gives you this function at any value of M .

c) Use the results from a) and b) to compute the halo mass function as

$$\frac{dn(z, M)}{d \ln M} = f(\sigma) \frac{\bar{\rho}_m}{M} \frac{d \ln \sigma^{-1}}{d \ln M} \quad (4)$$

for the fit from Tinker et al. 2008 (<https://arxiv.org/abs/0803.2706>), i.e.

$$f(\sigma) = A \left[\left(\frac{\sigma}{b} \right)^{-a} + 1 \right] \exp \left[-\frac{c}{\sigma^2} \right] \quad (5)$$

and choose values for A, a, b, c that are appropriate for $\Delta = 200$ (see Tinker, Eq. 3 and Table 2.). Plot $dn/d \ln M$ versus M in the same range as in a), for $z = 0$ and $z = 1$. Is $f(\sigma)$ properly normalized? What about $g(\sigma)$ from Eq. C2 and Table C4 in Appendix C?

d) Integrate $dn/d \ln M$ in mass M for masses above $M_{\text{lim}} = 10^{14} M_{\odot}/h$ for various values of z and interpolate to finally obtain the number density $n(z)$ at any z :

$$n(z) = \int_{M_{\text{lim}}}^{\infty} d \ln M \frac{dn(z, M)}{d \ln M} \quad (6)$$

Plot $n(z)$ versus z , for $z = [0, 2]$.

e) Finally, integrate $n(z)$ in comoving volume $dV = \Delta \Omega dz D_A^2(z)/H(z)$, for $\Delta \Omega = 5000 \text{ deg}^2$ (convert $\text{deg}^2 \rightarrow \text{rad}^2$) to find the number $N(z_i)$ of halos in redshift bins of width $\Delta z = 0.1$:

$$N(z_i) = \Delta \Omega \int_{z_i}^{z_i + \Delta z} dz \frac{D_A^2(z)}{H(z)} n(z) \quad (7)$$

Plot $N(z_i)$ versus z_i for 20 bins in z_i , i.e. from 0 to 2.