4300430: Introdução à Cosmologia Física

Problem Set 1

(Due August 20, 2024)

1) Experimental Time-Dilation : (worth 3 points)

On October 1971, cesium beam clocks were flown on jet flights around the world twice (eastward and westward) and then compared with reference clocks at the US Naval Observatory. From the flight paths of each trip, and considering only the special relativistic (kinematic) effect, compute how much time the clock moving eastward should have lost/gained relative to the reference clocks. Repeat the computation for the clock moving westward.

Note: On top of the kinematical effect, there is also a larger time dilation due to a gravitational effect from General Relativity (see Problem Set 2). Both the kinematical and gravitational effects are comparable and necessary to explain the observed time gain/loss.

Suggestion: Read the original paper *J. Hafele, R. Keating, Science, Vol 177, No 4044 (1972), pp. 166-168*

2) E & B Fields : (worth 4 points)

a) Write out the $\mu = 0$ component of the covariant force equation $f^\mu = qU_\nu F^{\mu\nu}$ in terms of the particle energy, velocity and the **E** & **B** fields, and provide an interpretation for it.

b) Using the Lorentz transformations on $F_{\mu\nu}$, show how **E** and **B** transform under a boost along the x-axis.

c) Show that for a general tensor $B_{\mu\nu}$, the contraction $B^{\mu\nu}B_{\mu\nu}$ is a scalar. Apply this result to the electromagnetic field tensor $F_{\mu\nu}$ to obtain the scalar in this case.

d) The energy-momentum tensor for electromagnetism is

$$T^{\mu\nu}_{(\mathrm{EM})} = F^{\mu\lambda}F^{\nu}{}_{\lambda} - \frac{1}{4}\eta^{\mu\nu}F^{\lambda\sigma}F_{\lambda\sigma}$$

Compute $T^{00}_{(\mathrm{EM})}$ and $T^{0i}_{(\mathrm{EM})}$ in terms of **E** and **B**.

3) Dodelson 2.1 (worth 3 points)

In addition to this problem, express:

- The critical density today $\rho_c = 3H_0^2/8\pi G$ in units of $h^2 M_\odot \mathrm{Mpc}^{-3}$,
- c/H_0 in units of $h^{-1}\mathrm{Mpc}$.