4300430: Introdução à Cosmologia Física

Final Project: N-body Simulations

(2016)

In this project you will evolve particles according to their own collective gravitational potential, also accounting for the background expansion of the Universe, in a Particle-Mesh (PM) N-body simulation. You will simulate a box of size $L_{\rm box} = 256{\rm Mpc}/h$, containing $N_p = 128^3$ particles. You should use at least $N_g = 128^3$ grid points to compute the density field and gravitational potencial. More grid points (e.g. $N_g = 256^3$) will give you higher resolution, but will also require longer times to run the simulation.

The initial particle positions and velocities will be specified at $z = 49$ ($a = 0.02$) and you will evolve the simulation in N_{ts} time steps until $z = 0$ ($a = 1$). Download here the file with initial conditions for the particles. The first two lines have the initial redshift and scale factor. The other lines are in the format (x, y, z, v_x, v_y, v_z) containing the comoving coordinates and peculiar velocities for each particle. Positions are in units of Mpc$/h$ and velocities in Km/s. The simulation is for cosmological parameters $\Omega_b = 0.045$, $\Omega_{\rm DM} = 0.222$, $\Omega_{\rm DE} = 0.733$, $\Omega_\nu = 0$, $T_{\rm CMB} = 2.726{\rm K}$, $w = -1$, $h = 0.72$, $n_s = 1$, $\sigma_8 = 0.8$.

Use the Lecture Notes on N-body simulations by A. Kravtsov to follow the steps below:

Simulation Steps

- From the particle positions $\mathbf{x}_j^p$, with $j = 1..N_p$, define the density field $\rho(\mathbf{x}_i)$, with $i = 1..N_g$, for each of the N_g grid points, using the Cloud-in-Cell (CIC) interpolation. Define an overdensity field $\delta(\mathbf{x}_i) = [\rho(\mathbf{x}_i) - \bar{\rho}_m]/\bar{\rho}_m$. You may want to work in units where $\bar{\rho}_m = 1$ (see code units in the Lecture Notes).

- Fourier Transform $\delta(\mathbf{x}_i) \to \tilde{\delta}(\mathbf{k}_i)$.

- Compute the gravitational potential $\tilde{\Phi}(\mathbf{k}_i)$ in Fourier space from the Poisson equation and the appropriate Green's function for discrete transforms (see Lecture Notes).

- Inverse Fourier Transform $\tilde{\Phi}(\mathbf{k}_i) \to \Phi(\mathbf{x}_i)$. (*Hint*: Check that the transform sequence $\rho(\mathbf{x}_i) \to \tilde{\rho}(\mathbf{k}_i) \to \rho(\mathbf{x}_i)$ indeed produces the original $\rho(\mathbf{x}_i)$ you started with).
- Compute the gradient $\mathbf{a}(\mathbf{x}_i) = -\vec{\nabla}\Phi(\mathbf{x}_i)$ or acceleration at each of the N_g grid points.
- Interpolate the acceleration at grid points $\mathbf{a}(\mathbf{x}_i)$ onto the particle positions $\mathbf{a}(\mathbf{x}_j^p)$.
- Update particle velocities/positions (leap-frog) in a time step Δa, ($a_{k+1} = a_k + \Delta a$).
- Repeat process for N_{ts} time steps, until $a_{(k=N_{ts})} = 1$.

Analysis of Simulation

1) Output particle positions at selected time-steps (e.g. at $z = 49$ (initial conditions), at $z = 20, z = 10, z = 5, z = 2, z = 1, z = 0.5$ and $z = 0.0$) or, if you have lots of space, output at all time steps during the simulation above.

2) Plot the overdensity field of these particles at each scale factor a, in a 2D plot showing $\delta(x, y)$ in (x, y) coordinates for particles within a slice of $[z, z + \Delta z]$ for $\Delta z = 20$ Mpc/h. Here z is the z-component of the 3D vector $\mathbf{x}$. How does the overdensity evolve with time? Can you make a "movie"of this evolution by concatenating outputs? In the latter case, you would probably need to output particle positions (or the overdensity field) at all time-steps...

3) Recall that in each time step you compute $\tilde{\delta}(\mathbf{k}_i)$. Use this to compute and plot the power spectrum $P(k)$ as a function of k for $z = 49$, $z = 10$, $z = 0$. Compute $P(k)$ as

$$P(k) = \frac{1}{(2\pi)^3 V} \langle |\tilde{\delta}(k)|^2 \rangle = \frac{1}{(2\pi)^3 L_{\text{box}}^3} \frac{1}{N_k} \sum_{i=1}^{N_k} |\tilde{\delta}(k_i)|^2 \tag{1}$$

in bins of k. Here $k^2 = |\mathbf{k}|^2 = k_x^2 + k_y^2 + k_z^2$. The average is done for all N_k points whose k_i is within $[k, k + \Delta k]$. Compare your $P(k)$ to the `CAMB` linear spectrum $P_L(k)$ at $z = 49, 10, 0$.

4) Look for a student working on the Halo Model project, and ask him to send you his Halo Model (HM) power spectrum $P^{\text{HM}}(k)$ at $z = 0$. How does it compare to the $P(k)$ you found directly from your simulation?