

4300430: Introdução à Cosmologia Física

Final Project: Halo Model

(2016)

Halo Mass-function

In Problem Set 10, you computed the halo mass-function for a fit from (Tinker et al. 2008) as described in the main text of this paper. Now let us consider Tinker's alternative fit

$$\frac{dn(z, M_{vir})}{d \ln M_{vir}} = g(\sigma) \frac{\bar{\rho}_{m0}}{M_{vir}} \frac{d \ln \sigma^{-1}}{d \ln M_{vir}} \quad (1)$$

$$g(\sigma) = B \left[\left(\frac{\sigma}{e} \right)^{-d} + \sigma^{-f} \right] \exp \left[-\frac{g}{\sigma^2} \right] \quad (2)$$

where instead we use the function $g(\sigma)$ from Appendix C in Tinker's paper (why do we do this?). Let us set parameter values for parameters B, d, e, f, g which are appropriate for $\Delta = \Delta_c/\Omega_m$ (computed from Eqs. 10-12 below, which sets the mass to be the virial mass $M = M_{vir}$). For $\Omega_m = 0.23$, $\Delta \approx 397$, which you may approximate as $\Delta = 400$ for purposes of using Tinker's fits. Here $\sigma^2(M_{vir})$

$$\sigma^2(z, R) = G^2(z) \int \frac{k^2 dk}{2\pi^2} |W(kR)|^2 P_L(k) \quad (3)$$

where $M_{vir} = \bar{\rho}_{m0} 4\pi R^3/3$, $\bar{\rho}_{m0} = \Omega_m \rho_{crit,0}$, $P_L(k)$ is the linear power spectrum at redshift $z = 0$, $G(z) = D(z)/D(z=0)$ is the growth function normalized to its value today, and

$$W(kR) = \frac{3}{k^2 R^2} \left[\frac{\sin(kR)}{kR} - \cos(kR) \right] \quad (4)$$

is the Fourier Transform of a spherical top-hat window of radius R .

For a fiducial cosmology, compute the halo mass-function above and plot it versus M_{vir} for $z = 0$ and $z = 1$ in the range $M_{vir} = [10^{12}, 10^{16}] M_\odot/h$. Use log scale in both axes.

Check that the mass function is properly normalized i.e. that $-\int [g(\sigma)/\sigma] d\sigma = 1$.

Halo Bias

For the fiducial cosmology, compute the halo bias for the fit from [Tinker et al. 2010](#):

$$b(z, M) = 1 - A \frac{\nu^a}{\nu^a + \delta_c^a} + B\nu^b + C\nu^c \quad (5)$$

where $\nu = \delta_c/\sigma$. Again, for consistency use values of A, a, B, b, C, c which are appropriate for $\Delta = \Delta_c/\Omega_m$ (≈ 400 for $\Omega_m = 0.23$), i.e. $M = M_{vir}$. See Tinker's Eqs. 6 and 7 and Table 2. Notice that the paper for Tinker's bias (2010) is different from that for the mass-function (2008)!

Plot $b(z, M_{vir})$ versus M_{vir} for $z = 0$ and $z = 1$ in the range $M_{vir} = [10^{12}, 10^{16}]M_\odot/h$. Use log scale in the M_{vir} axis and *linear* scale for $b(M_{vir})$.

Check that the bias function above is properly normalized, i.e. $\int [g(\nu)b(\nu)/\nu]d\nu = 1$.

If the mass-function and/or bias are *not* normalized appropriately, you will likely obtain incorrect results for the 2-halo term in Eq. 19 below (which will not properly approach the linear spectrum at large scales). The 1-halo term in Eq. 18 will also be affected.

Halo Profile

For the fiducial cosmology, compute the halo profile for the fit of NFW:

$$\rho(r|M_{vir}, z) = \frac{\rho_s}{cr/r_{vir}(1 + cr/r_{vir})^2} \quad (6)$$

for halos of mass $M_{vir} = 10^{14}$ and $10^{15} M_\odot/h$, at redshifts $z = 0$ and $z = 1$. Plot $\rho(r)$ versus r for these 4 cases in the same plot with log scale in both axes. Notice that ρ_s , c and r_{vir} are functions of M_{vir} and z . In order to determine these 3 quantities for a given M_{vir} and z , you need the formulae sequence below:

$$E^2(z) = \Omega_m(1+z)^3 + \Omega_{DE}(1+z)^{3(1+w)}, \quad (7)$$

$$\rho_{crit,0} = 2.775 \times 10^{11} h^2 M_\odot \text{Mpc}^{-3}, \quad (8)$$

$$\rho_{crit}(z) = \rho_{crit,0} E^2(z), \quad (9)$$

$$\omega_m(z) = \Omega_m(1+z)^3 / E^2(z), \quad (10)$$

$$x = \omega_m(z) - 1, \quad (11)$$

$$\Delta_c = 18\pi^2 + 82x - 39x^2, \quad (\text{Bryan \& Norman 1997}) \quad (12)$$

$$\Delta_c = \frac{3M_{vir}}{\rho_{crit}(z)4\pi r_{vir}^3} \rightarrow \text{Find } r_{vir}, \quad (13)$$

$$\nu(M^*) = 1 \rightarrow \text{Find } M^*, \quad (14)$$

$$c(M_{vir}, z) = \frac{9}{1+z} \left[\frac{M_{vir}}{M^*} \right]^{-0.13} \quad (\text{Bullock et al. 2001}) \rightarrow \text{Find } c, \quad (15)$$

$$M_{vir} = 4\pi\rho_s \frac{r_{vir}^3}{c^3} \left[\ln(1+c) - \frac{c}{1+c} \right] \rightarrow \text{Find } \rho_s, \quad (16)$$

Finally compute numerically or analytically (see [Cooray & Sheth 2002](#)) the Fourier Transform for the spherically symmetric halo profile at $z = 0$:

$$u(k|M_{vir}) = \int_0^{r_{vir}} dr 4\pi r^2 \frac{\sin kr}{kr} \frac{\rho(r|M_{vir})}{M_{vir}} \quad (17)$$

Plot $u(k|M_{vir})$ versus k at $z = 0$, for $M_{vir} = 10^{10}, 10^{11}, 10^{12}, 10^{13}, 10^{14}, 10^{15}, 10^{16} M_{\odot}/h$. Show all cases in the same plot, and use log scale in both axes. Do you find something close to a similar to Fig. 9 in [Cooray & Sheth 2002](#) ?

Make sure $u(k|M_{vir}) \rightarrow 1$ as $k \rightarrow 0$ for all values of M_{vir} .

Halo Model: Nonlinear Power Spectrum

Combine the Halo Model ingredients you found before to compute the Halo-Model power spectrum at $z = 0$ with the formulae sequence:

$$P^{1h}(k) = \int d \ln M \frac{M^2}{\bar{\rho}_{m0}^2} \frac{dn}{d \ln M} |u(k|M)|^2 \quad (18)$$

$$P^{2h}(k) = \left[\int d \ln M \frac{M}{\bar{\rho}_{m0}} \frac{dn}{d \ln M} b(M) u(k|M) \right]^2 P_L(k) \quad (19)$$

$$P(k) = P^{1h}(k) + P^{2h}(k) \quad (20)$$

Note that as $k \rightarrow 0$, $u(k|M) \rightarrow 1$, so in Eq. 19 the term within []'s $\rightarrow 1$ (due to the halo bias normalization condition), and therefore $P^{2h}(k) \rightarrow P_L(k)$. However for finite and large k , $P^{2h}(k) \neq P_L(k)$. Plot in the same figure using different line types and colors:

- $P_L(k)$ (linear spectrum),
- $P^{1h}(k)$ (1-halo term),
- $P^{2h}(k)$ (2-halo term),
- $P(k)$ (total halo model spectrum),
- $P^{HF}(k)$ (the non-linear spectrum from `halofit` (Takahashi 2012). See nonlinear options inside `CAMB`).

Use log scale in both axes and set the range $k = [10^{-3}, 10^1]h/\text{Mpc}$ (change the value of k_{max} in `CAMB` if necessary). Below is a sample plot of $P_L(k)$ and $P^{HF}(k)$ (first and last in the list above). The idea is for you to fill a plot like this with the other terms.

How does your $P(k)$ compare to $P^{HF}(k)$?

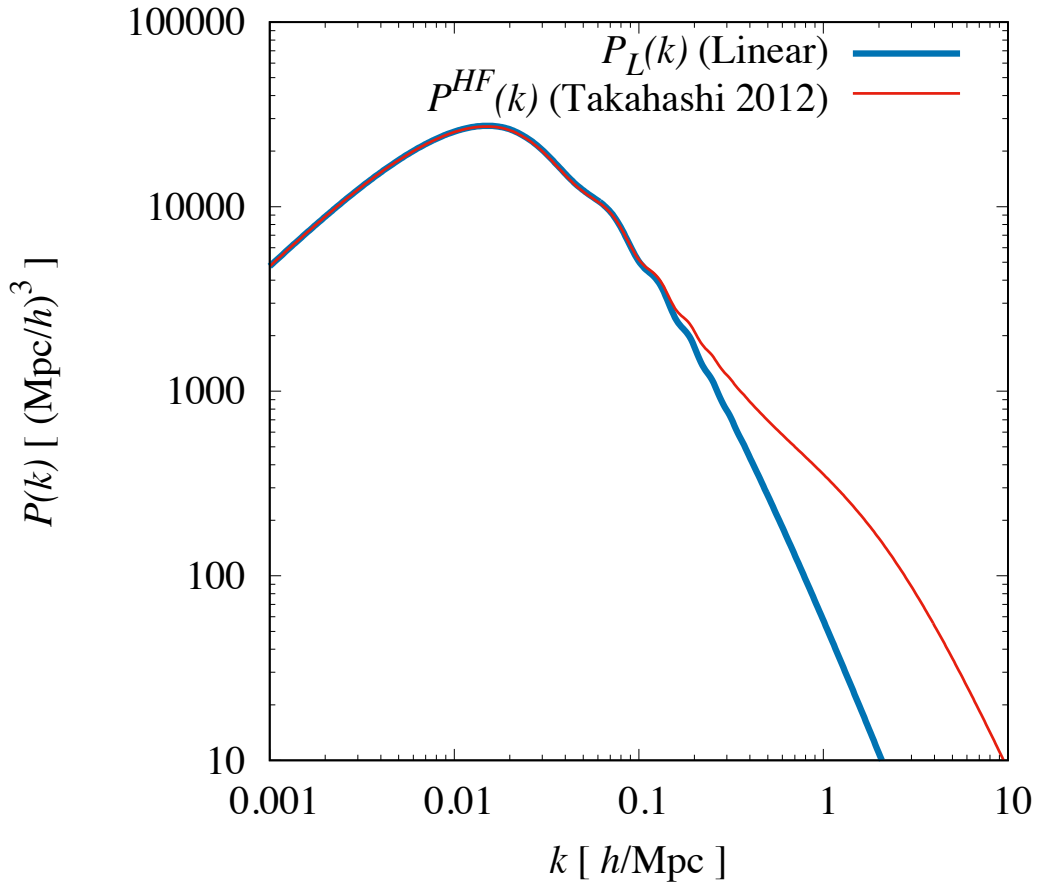


Figura 1: The thick blue line indicates the linear power spectrum $P_L(k)$ from CAMB. The thin red line is a fit to simulations for the nonlinear power spectrum $P^{HF}(k)$ from the `halofit` code by Takahashi et al. 2012 (<https://arxiv.org/abs/1208.2701>). Hopefully your total Halo Model spectrum $P(k)$ will be similar to the `halofit` nonlinear spectrum.