Comment on "Dynamical Foundations of Nonextensive Statistical Mechanics"

Recently, Beck [1] discussed a class of Brownian motions described by a Langevin equation (LE) of the form

$$\dot{u} = -\gamma u + \sigma L(t), \tag{1}$$

where u is the velocity, γ is the friction, and $L(t)$ is a Gaussian noise with strength σ. He allows the ratio $\beta = \gamma/\sigma^2$ to change in such a way that β is a Γ distribution. He finds the rms of β to be $\propto 1/n$. Based on this assumption, he concludes that the velocity distribution function (VDF) is

$$P(u) \propto [1 + \tilde{\beta}(q-1)u^2/2]^{1/(1-q)}, \tag{2}$$

where $q = 1 + 2/(n+1)$, and n is the number of velocity degrees of freedom.

There are several controversial statements and assumptions in this work. The purpose of this Comment is to clarify some of these. The LE and its generalizations must satisfy the equipartition theorem and the fluctuation-dissipation theorem (FDT). That establishes a fixed value for $\beta = \gamma/\sigma^2$, a result that Beck uses as an assumption. A basic assumption in [1] is to let inverse generalized temperature β fluctuate in time and space. However, one cannot allow the same position in space to have a distribution of temperatures. This does not produce stable distributions, neither Gaussians nor power laws. Beck goes on to assume that the β fluctuations are due to fluctuations in noise strength and/or the friction. Time-changing friction is usually generated by colored noise, which induces memory with a nonzero correlation time. For these systems, the generalized Langevin equation (GLE) will be the proper way to treat the problem [2–4]. Both the LE and GLE have well-defined detailed balance, i.e., FDT, and this is not clear in Beck's work as well as the origin of a fluctuating friction.

Later in his work, the author introduces a new LE, his Eq. (13), with a nonlinear friction term. For this, he provides a new VDF, Eq. (15), whose velocity dependency is the same as in the linear case, Eq. (6), apart from a scaling of the velocity $u \to |u|^\alpha$. The claim that the VDF will take this form is a very strong statement which is left completely unproven in [1].

In order to connect his work to experimental data, Beck uses two values of $\alpha \approx 0.90$ and 0.92 (i.e., two distinct physical laws) to get two values of q that fit two different experiments in turbulence. With both the average of β, n, and α [or equivalently q through Eq. (17), an equation derived for one-dimensional systems] as adjustable parameters, good fits should result. In the case of the experiment in [4], Beck supposes the statistics is done for a system of one particle in three dimensions, $n = 3$, which is a very restrictive condition, and which produces enormous fluctuations. However, we note that taking three particles in one dimension would lead to the same statistics (assuming α to remain essentially unchanged) for the velocities. But, it is well known that turbulence depends strongly on dimensionality [6]. Lastly, we note that the fits to the experiments of [4] have been recently questioned by refining those data [7].

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