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Morphology of growth by random walk deposition

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Abstract

We simulate a deposition model with a break of symmetry induced by a point source. We find that Tsallis anomalous distribution for random walks $n(x) = N(A)/[1 + b(q-1)x^2]^{q/(q-1)}$ produces a good fit to the data. We obtain the mean square displacement $\langle x^2 \rangle$ and the total number of deposited particles N, and compare them to the Gaussian case. Our main conclusions are twofold: first, the parameter q is size dependent; second, long range correlations imply in a violation of the law of great numbers. © 2001 Elsevier Science B.V. All rights reserved.

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The problem of describing the growth of a system is one of the basic questions of dynamics. In particular, the study of far from equilibrium kinetics of interfaces has found a wide range of applications in the growth of bacterial colonies and crystalline solids [1], etching of a crystalline solid by a liquid [2] and stress in rough surfaces in contact [3]. Those have enormous applications in physics [1], chemistry [4], geophysics [3] and many other fields of science [1].

In this paper, we propose a simple model for deposition from a single source of particles at a height H and at a position $i = 0$, i.e., at the coordinate $(0, H)$ of the xy plane. The model is atomistic in the sense that the particles have unit size and each discrete motion is a composition of a step down and a horizontal random step

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either to the left or to the right. In the beginning, the particles fall in diagonal with equal probability to the left or to the right. However, as deposition evolves, one of the particles may have its path blocked by a previously deposited particle, thus falling down one step vertically. The motion continues until the particle reaches the bottom of the surface at the position $(i, 1)$ or until it reaches the top of another particle at the position $(i, n_i + 1)$ $(i = -H + 1, \ldots, 0, \ldots, H - 1)$, where $n_i = 0, 1, \ldots, H - 1$ is the number of deposited particles at the site i. Consequently, the number of deposited particles is increased by one, $n_i \rightarrow n_i + 1$, and the particle stops. The process continues until H particles are deposited at the column $i = 0$.

Now that our cellular automata model has been established, we can study the growth process as a diffusion process. Since the particles have unit size, the height h_i of the column is equal to the number of particles, i.e., $h_i = n_i$. As well, for large numbers, we may use the continuous notation $n_i \rightarrow n(x) = h(x)$.

We shall consider here two possibilities of distributions to fit the curve: First, the Gaussian one:

$$n(x) = \frac{N}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{x^2}{2\sigma^2}} , \tag{1}$$

where $\sigma^2 = \langle x^2 \rangle$ is the mean square displacement and N the number of deposited particles. Second, the Tsallis distributions [5–10]:

$$n(x) = N \frac{A}{[1 + b(q-1)x^2]^{\alpha}} , \tag{2}$$

where $\alpha = q/(q - 1)$. We shall call the random walks described by Eq. (1) as normal random walk (NRW) and any other as anomalous or generalized random walk, GRW. One should notice that as $q \rightarrow 1$ the right hand side of Eq. (2) becomes a Gaussian and we expect that the usual diffusion holds.

In Fig. 1, we plot the number of deposited particles $n(x)$ as a function of the position x. The particles fall from a height $H = 700$ and we take an average over 1000 experiments. The number of particles deposited in each curve are: curve (a) 8102 particles; curve (b) 16 205 particles; curve (c) 24 307 particles; and curve (d) 32 046 particles. Those curves show the evolution of the height distributions or of the density of deposited particles $n(x)$. As the number of particles deposited grows, the curves behave more and more like a power law with 1(a) $q = 1.06$, 1(b) $q = 1.17$; 1(c) $q = 1.29$; and 1(d) $q = 1.63$. This is an apparent contradiction to the law of great numbers which states that the larger a distribution is, the closer to a Gaussian it becomes. It is possible to fit Gaussians to curves 1(a) to 1(d). However the curves obtained by using the Eq. (2) do it better (they have a smaller standard deviation than the Gaussians).

The evolution of the pile heights towards a power law is a measure of the dependence of the system on its history. A particle can fill a position (i, h_i) only if its step is not blocked. As more particles are deposited the increasing correlation between the particle and its neighbors increases the possibility of a frustrated motion. Thus the deviation from the law of great numbers is not a failure of Tsallis approach, it is a consequence

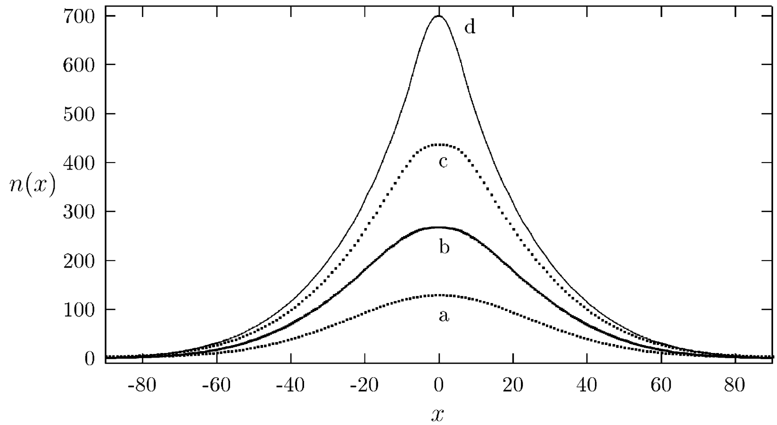

Fig. 1. We plot the density of deposited particles $n(x)$ as a function of x. The particles falls from a source at height $H = 700$. Every curve is an average of 1000 experiments. The number of deposited particles are: (a) 8102; (b) 16 205; (c) 24 307; (d) The curve saturates with 32 046 particles. As the piles grow the curves get away from Gaussian, violating the law of the great numbers.

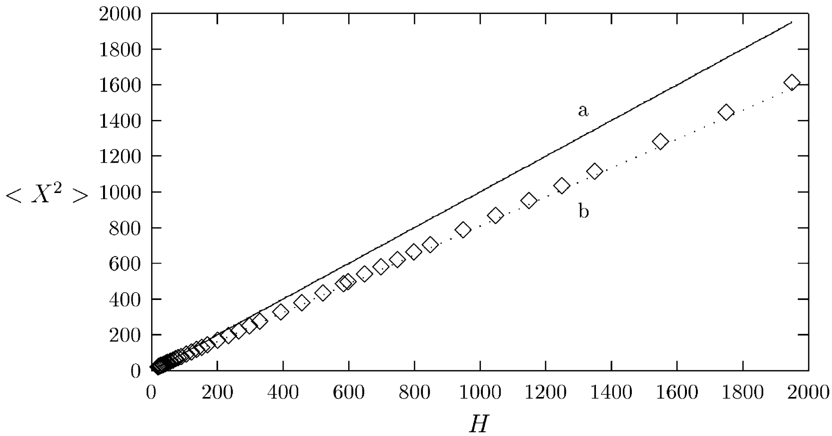

Fig. 2. We plot the mean square displacement $\langle x^2 \rangle$ as a function of height H for the saturated deposition (a) The theoretical value for a Gaussian distribution; (b) The data are from the simulation and the line is its fitting.

of the long range correlation. Since q measures the degree of non-additivity, increasing the number of particles and its correlation will make q deviate from 1. The law of the great numbers is for uncorrelated distributions. It is worthwhile to remark that Lèvy [11–13] distributions have infinite $\sigma = \sqrt{\langle x^2 \rangle}$ for finite H, consequently we shall rule out the possibility of Lèvy distributions. Notice as well that the set of experiments described on Figs. 2–4 are better described by the Eq. (2).

This system does not present translational symmetry. Consequently, it does not satisfy the Family–Viesek scaling relation [14]. Recently [2], we used a combination of Gaussians and Lèvy curves to fit the heights fluctuations for corrosion in a solid–solid

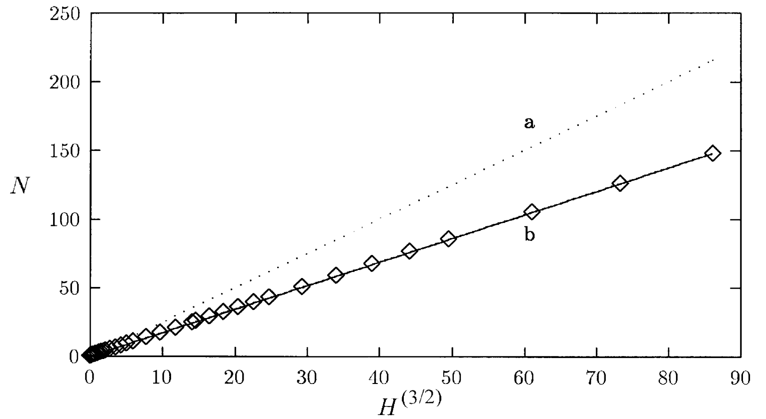

Fig. 3. The total number of deposited particles at the saturation N as a function of the height $H^{(3/2)}$. Both axes are divided by 1000. (a) The theoretical value for the Gaussian distribution; (b) The line is the fit of the data.

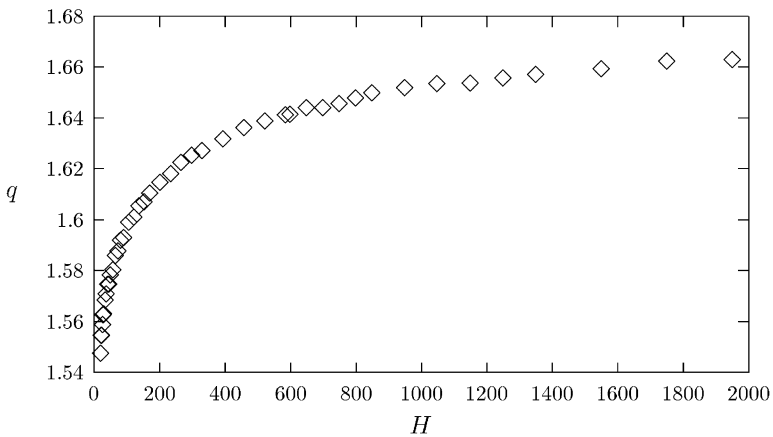

Fig. 4. The parameter q as a function of the height H. As the number of deposited particles grows the q increases and the curve deviates from a Gaussian and becomes more and more a power law. This too confirms the result of curve Fig. 1 for different heights.

surface. Since those satisfy the Family–Viesek scaling relation, they are not appropriate to describe our distributions.

We notice as well in Fig. 1 that the curves start out as Gaussians and change their shapes as the pile grows. This is a direct consequence of the "collisions" with another particles. The first characteristic of a normal random walk is that the mean square displacement is just $\sigma^2 = \langle x^2 \rangle = H$, i.e., the number of steps. Consequently that corresponds to the first idea to be investigated.

In Fig. 2, we compute $\langle x^2 \rangle$ versus H for a large number of saturated distributions such as that of Fig. 1(d). The result can be cast in the form

$$\langle x^2 \rangle = a_{\mathrm{H}} H \,, \tag{3}$$

where $a_{\mathrm{H}} = 0.82$. Notice that we would expect $a_{\mathrm{H}} < 1$ since we predict that some particles will be frustrated in their attempt to do a NRW.

The surprising result is that Eq. (3) is similar to that obtained using a Gaussian distribution of the form in Eq. (1). An easy way to compare the Gaussians, Eq. (1), with the power law, Eq. (2), is to notice that for Gaussians the total number of deposited particles for a given H is $N = \sqrt{2\pi}H^{3/2}$. This is readily obtained from Eq. (1) if we make $n(0) = H$.

In Fig. 3, we plot the total number of particles N as function of $H^{3/2}$. The axes are divided by 1000. Fig. 3(a) is a Gaussian while Fig. 3(b) is the fit to the experiments. The result of this figure can be summarized as

$$N = bH^{3/2} . \tag{4}$$

Here $b \approx 1.72$ while for the Gaussians $b = \sqrt{2\pi} \approx 2.50$.

There is no doubt that the power law, Eq. (1), will fit better the set of simulations than the Gaussians. However, there is a price to pay, mainly the parameter q becomes size dependent as can be seen in Fig. 4, where we plot the parameter q as a function of the height H. Data are the same as for Figs. 2 and 3. Notice that q is an increasing function of H, that is, an increasing function of the number of deposited particles. Since we are working with a saturated distribution, the evolution of q is not so drastic as in the experiment described in Fig. 1. However, q is clearly an increasing function of the number of particles. As H grows q is governed by the law $q = q_{\max} - c/H^{\gamma}$, with $q_{\max} = 1.82 \pm 0.09$, $\gamma = 0.11 \pm 0.06$, and $c = 0.38 \pm 0.02$. It is important to notice that some of the authors [10] considered the evolution of the Tsallis distributions, Eq. (2), and demonstrated that they are scaling dependent, consequently we would expect $q = q(N)$.

Our simulations are in the range $20 < H < 2000$. For values of H larger than those discussed here the computer time becomes prohibitive. However, finite effects starts to disappear for $H > 5000$. The fact that the main characteristic of the Gaussians are kept, i.e., Eq. (3) and Eq. (4), one may think about an effective average number of steps in the vertical, $N_{\mathrm{SV}} = a_{\mathrm{V}}H$, and an effective average number of steps in the horizontal, $N_{\mathrm{SH}} = a_{\mathrm{H}}H$. For Gaussians, obviously $a_{\mathrm{V}} = a_{\mathrm{H}} = 1$. Since, in general, a_{H} is more complex, we try to understand first the average number of steps in the vertical. For a column of height h_i, the particle will take $H - h_i$ steps. Consequently, the number of steps for each column will be $(2(H-1) - h_i)h_i/2$ and the total number of steps per particle in the vertical will be

$$N_{\mathrm{SV}} = \frac{1}{2N} \sum_{i=-H+1}^{H-1} (2(H-1) - h_i)h_i = \left(H - \frac{1}{2}\right) - \frac{1}{2N} \sum h_i^2 . \tag{5}$$

Hence,

$$a_{\mathrm{V}} = 1 - \frac{1}{2H} - \frac{\Gamma(2\alpha - 1/2)\Gamma(\alpha)}{2\Gamma(2\alpha)\Gamma(\alpha - 1/2)} . \tag{6}$$

The first term at the right of Eq. (6) is a Gaussian, the second is a finite size effect, and the last one is the correction we want. Nevertheless its form is almost independent of H. For the range of heights we worked with, $a_{\mathrm{V}} \approx 0.7$. One main question that arises is why $N_{\mathrm{SH}} > N_{\mathrm{SV}}$? The number of steps in the horizontal is the combination of two competing effects. First, the number of vertical steps decreases because the particles find the column filled with some particles as in Eq. (6). Second, the frustration in the horizontal is more likely to occur when the particles approach the origin than when they get away from it. That favors large $\langle x^2 \rangle$. Those competitors give the surprising results we obtain, mainly $\langle x^2 \rangle \propto H$ and $N \propto H^{3/2}$ which are the same result, except for the coefficients, obtained exactly for Gaussians. This is a surprising result since for a GRW one would expect for example $\langle x^2 \rangle \propto H^{\mu}$, with $1 < \mu < 2$.

In conclusion, we study a deposition model that occurs when a single source is sited at the origin. We compare the result when the number of deposited particles in a position $n(x)$ is either a Gaussian or a Tsallis distribution. From Gaussians, without frustration, it is possible to obtain the mean square displacement and the total number of particles as a power of the type cH^{α} . Surprisingly the computer experiment agrees with that and so does the result obtained by the power law distribution $n(x)$. Thus, the exponents are the same for both, and the coefficient is more precise for the Tsallis distribution. Thus, Tsallis distribution seems to describe the statistical features of systems in which the presence of long-range correlations implies in a violation of the law of great numbers.

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