Exemplo Não abeliano (SU(2))

$\phi(x)$ se ~~transforma~~ na rep. fundamental de SU(2) (é um "vetor" de SU(2))

Dobleto $\phi(x) = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}$ ϕ_1 e ϕ_2 complexos

$$\mathcal{L} = \frac{1}{2}(D_\mu \phi)^\dagger D^\mu \phi - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu}_a - V(\phi)$$

$$D_\mu \phi = \left(\partial_\mu - i g A_\mu^a t^a\right)\phi$$

Se $\langle \phi \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$

$$\Rightarrow (D_\mu \phi)^\dagger D^\mu \phi = \frac{g^2}{2}\begin{pmatrix} 0 & v \end{pmatrix} t^a t^b \begin{pmatrix} 0 \\ v \end{pmatrix} A_\mu^a A^{b\mu}$$

$+ \ldots$

$$\{t^a, t^b\} = \frac{1}{2}\delta^{ab}$$

$$t^a t^b + t^b t^a = \frac{1}{2}\delta^{ab}$$

$$\to t^a t^b A_\mu^a A^{b\mu} = \left(\frac{t^a t^b A_\mu^b A^{\mu a}}{2} + \frac{t^b t^a A^a A^b}{2}\right)$$

$$= A^a A^b \frac{1}{2} \cdot \frac{1}{2}\delta^{ab}$$

$\Rightarrow$

$$= \cdots \frac{g^2 v^2}{8} A_\mu^a A^{a\mu}$$

$\Rightarrow$ Todos recebem

$$\boxed{m_A = \frac{gv}{2}}$$

SU(2) é completamente quebrado

$$V(\Phi^{+}\Phi) = -\frac{m^2}{2}\Phi^{+}\Phi + \frac{\lambda}{4}(\Phi^{+}\Phi)^2$$

$$\Rightarrow \frac{\partial V}{\partial \Phi^{+}\Phi} = -\frac{m^2}{2} + \frac{\lambda}{2}\underbrace{\langle \Phi^{+}\Phi \rangle_0}_{v^2} = 0$$

$$\Rightarrow \boxed{v^2 = \frac{m^2}{\lambda}}$$

$\Rightarrow$ Setor escalar :

$$\Rightarrow \Phi(x) = \Phi'(x) + \langle \Phi \rangle_0$$

$$= \Phi'(x) + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Rightarrow \begin{cases} \phi_1(x) = \phi_1' = \xi_1 \\ \phi_2(x) = \phi_2' + \frac{v}{\sqrt{2}} \end{cases}$$

Massas?

$$\Phi^\dagger\Phi = \left(\Phi^{\dagger\prime} + \langle\Phi^\dagger\rangle_0\right)\left(\Phi' + \langle\Phi\rangle_0\right)$$

$$= \Phi^{\dagger\prime}\Phi' + \langle\Phi^\dagger\rangle_0\Phi' + \Phi^{\dagger\prime}\langle\Phi\rangle_0 + \langle\Phi^\dagger\rangle_0\langle\Phi\rangle_0$$

$$\frac{m^2}{2}\Phi^\dagger\Phi = \left[\cancel{\phi_1'^2} + \cancel{\phi_2'^2} + \frac{v}{\sqrt{2}}\left(\phi_2' + \phi_2'^\dagger\right) + v^2\right]\frac{m^2}{2}$$

$$\left(\Phi^\dagger\Phi\right)^2 \frac{\lambda}{4} = \frac{\lambda}{4}\left[\cancel{2v^2\phi_1'^2} + \cancel{2v^2\phi_2'^2} + \frac{v^2}{2}\left(\phi_2' + \phi_2'^\dagger\right)^2 + \dots\right]$$

⇒ Só uma combinação é massiva

$$\frac{\phi_2' + \phi_2'^\dagger}{\sqrt{2}} = \sqrt{2}\,\mathrm{Re}[\phi_2']$$

→ Podemos escrever

$$\phi = \begin{pmatrix} B+iC \\ h+iA \end{pmatrix}$$

⇒ A, B, C são Boson de Goldstone

Só h tem massa

Quebra Espontânea de Simetria Local

Mecanismo de Higgs:

A24

Simetria U(1) Local →

$$\mathcal{L} = \frac{1}{2}(D_\mu\phi)^\dagger D^\mu\phi - V(\phi^*\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$D_\mu = \partial_\mu + ieA_\mu$$

$$\begin{cases} \phi(x) \longrightarrow e^{i\alpha(x)}\phi(x) \\ A_\mu(x) \to A_\mu - \frac{1}{e}\partial_\mu\alpha(x) \end{cases}$$

$$V(\phi^*\phi) = \frac{\mu^2}{2}\phi^*\phi + \frac{\lambda}{4}\phi^*\phi$$

Quebra espontânea de simetria

(2)

Se $\mu^2 < 0 \Rightarrow m^2 = -\mu^2 > 0$

$$V(\phi^\dagger\phi) = -\frac{m^2}{2}\phi^\dagger\phi + \frac{\lambda}{4}(\phi^\dagger\phi)^2$$

Mínimo para $\phi_0 \neq 0$

$$\frac{\partial V(\phi^\dagger\phi)}{\partial(\phi^\dagger\phi)} = -\frac{m^2}{2} + \frac{\lambda}{2}(\phi^\dagger\phi) = 0$$

$$\Rightarrow \boxed{\phi_0^\dagger\phi_0 = \frac{m^2}{\lambda}}$$

$\Rightarrow$ Choose

$$\phi(x) = \phi_0 + \eta(x) + i\xi(x) \qquad \langle\phi(x)\rangle = \phi_0$$