Outra Interpretação da Ação Efetiva

Vimos que

$$Z[J] = e^{iW[J]}$$

onde a ação efetiva é definida como

$$\Gamma[\phi_c] = W[J] - \int d^4x\, J(x)\phi_c(x)$$

Em geral, vimos que

$$G^{(m)}(x_1 \dots x_m) = \frac{1}{Z[0]} (-i)^m \frac{\delta^m}{\delta J(x_1) \dots \delta J(x_m)} Z[J] \Big|_{J=0}$$

Permite a expressão

$$Z[J] = Z[0] \sum_{m=0}^{\infty} \frac{i^m}{m!} \int dx_1 \dots dx_m\, G^{(m)}(x_1 \dots x_m) J(x_1) \dots J(x_m)$$

Do mesmo jeito, vimos que

$$iW[J] = \sum_{m=0}^{\infty} \frac{i^m}{m!} \int dx_1 \dots dx_m\, G_c^{(m)}(x_1 \dots x_m) J(x_1) \dots J(x_m)$$

onde $G_c^{(m)}(x_1 \dots x_m)$ é a função de Green de m pontos

<u>conectada</u>

(i)

$$\int_0^{\Lambda} \frac{dk_E}{k_E} \left(V''(\phi)\right)^2$$

$$\left\{ \Gamma[\phi_c] = W[J] - J(x)\,\phi_c(x) \right\}$$

$$\left\{ \begin{array}{l} \dfrac{\delta W[J]}{\delta J(x)} = \phi_c(x) \\[2ex] \dfrac{\delta \Gamma[\phi_c]}{\delta \Phi_c(y)} = -J(y) \end{array} \right.$$

$$\frac{\delta^2 W[J]}{\delta J(y)\,\delta J(x)} = \frac{\delta \Phi_c(x)}{\delta J(y)}$$

$$\frac{\delta^2 W}{\delta J\,\delta J} = \left(\frac{\delta^2 \Gamma}{\delta \phi_c\,\delta \phi_c}\right)^{-1}$$

$$\frac{\delta}{\delta J(y)} \frac{\delta \Gamma[\phi_c]}{\delta \Phi_c(x)} = -\delta(x-y)$$

$$\delta(x-y) = -\int d^4z\, \frac{\delta \Phi_c(z)}{\delta J(y)} \frac{\delta^2 \Gamma}{\delta \Phi_c(z)\,\delta \Phi_c(x)} = \int d^4z\, \underline{\frac{\delta^2 W[J]}{\delta J(y)\,\delta J(x)}}\, \frac{\delta^2 \Gamma}{\delta\phi\,\delta\phi}$$

Higher deriv. (ii)

$$\frac{\delta}{\delta J(z)} = \int d^4w \, \frac{\delta \Phi_c(w)}{\delta J(z)} \frac{\delta}{\delta \Phi_c(w)}$$

$$\frac{\delta}{\delta J(z)} = -i \int d^4w \, D(z,w) \frac{\delta}{\delta \Phi_c(w)}$$

$$\Rightarrow \frac{\delta^3 W[J]}{\delta J_x \, \delta J_y \, \delta J_z} = -i \int d^4w \, D(z,w) \frac{\delta}{\delta \Phi_c(w)} \overbrace{\left(\frac{\delta^2 \Gamma[\Phi_c]}{\delta \Phi_x \, \delta \Phi_y} \right)^{-1}}^{\text{inv. de } \frac{\delta^2 W}{\delta J \delta J}!}$$

MAS $\frac{\partial M}{\partial \alpha}$ / $M M^{-1} = \mathbb{1}$

$$\left(\frac{\partial M}{\partial \alpha} \right) M^{-1} + M \frac{\partial M^{-1}}{\partial \alpha} = 0 \Rightarrow M \frac{\partial M^{-1}}{\partial \alpha} = -\frac{\partial M}{\partial \alpha} M^{-1}$$

$$\frac{\partial M^{-1}}{\partial \alpha} = - M^{-1} \frac{\partial M}{\partial \alpha} M^{-1}$$

$$= i \int d^4w \, D(z,w) \int d^4u \int d^4v \, D(x,u) (-i) \underbrace{\frac{\delta^3 \Gamma[\Phi_c]}{\delta \Phi_c(u) \, \delta \Phi_c(v) \, \delta \Phi_c(w)}}_{\substack{\text{1PI} \\ \text{3-point} \\ \text{function!}}} (-i) D(y,v)$$

A ação efetiva é também uma funcional Geratriz: gera as funções de correlação <u>1PI</u>

$$i\Gamma(\phi_c] = \sum_m \frac{1}{m!} \int dx_1 \ldots dx_m \, \Gamma^{(m)}(x_1 \ldots x_m)\, \phi_c(x_1) \ldots \phi_c(x_m)$$

<u>Tal que</u>:

$$\frac{\delta^m \, i\Gamma[\phi_c]}{\delta\phi_c(x_1) \ldots \delta\phi_c(x_m)} = \Gamma^m(x_1 \ldots x_m) \equiv G^{(m)}_{1PI}(x_1 \ldots x_m)$$

<u>Provar</u> <u>depois</u> (Peskin 11.5)

<u>No espaço de momentos</u>

$$\Gamma^{(m)}(x_1 \ldots x_m) = \int \frac{d^4k_1}{(2\pi)^4} \cdots \frac{d^4k_m}{(2\pi)^4} \, e^{i(k_1 \cdot x_1 + \ldots k_m \cdot x_m)} \, \Gamma^{(m)}(k_1 \ldots k_m)$$

$$\times (2\pi)^4 \, \delta^{(4)}(k_1 + \ldots + k_m)$$

Então, a ação efetiva é

$$i\Gamma[\phi_c] = \sum_m \frac{1}{m!} \int dx_1 \dots dx_m \int \frac{d^4k_1}{(2\pi)^4} \cdots \frac{d^4k_m}{(2\pi)^4} \times (2\pi)^4 \delta^{(4)}(k_1 + \dots + k_m)$$

$$\times e^{i(k_1 \cdot x_1 + \dots + k_m \cdot x_m)} \Gamma^{(m)}_{(k_1 \dots k_m)} \phi_c(x_1) \cdots \phi(x_m)$$

Por outra parte, a ação efetiva expandida em potências do momento é

$$\Gamma[\phi_c] = \int d^4x \left\{ -V_{eff}(\phi_c) + \frac{1}{2}\partial_\mu \phi_c \partial^\mu \phi_c Z(\phi_c) + \dots \right\}$$

onde as potências do momento correspondem com derivadas de ϕ_c é os ... se referem a potências ainda maiores do momento (mais derivadas).

Isto quer dizer que os $\Gamma^{(m)}_{(k_1, \dots, k_m)}$ pode ser expandido em potências do momento e que os termos $\Gamma^{(m)}_{(0, \dots, 0)}$ corresponderam (em forma) à $V_{eff}(\phi_c)$ dado que esse termo não tem derivadas.

$$i\Gamma[\phi_c] = \sum_m \frac{1}{m!}\int dx_1 \ldots dx_m \int \frac{dk_1}{(2\pi)^4} \cdots \frac{dk_m}{(2\pi)^4}$$

$$\times \int dx\, e^{i(k_1+\cdots+k_m)\cdot x}\; e^{i(k_1\cdot x_1+\cdots+k_m\cdot x_m)}$$

$$\times \left\{ \Gamma^{(m)}_{(0,\cdots,0)}\, \phi_c(x_1)\cdots\phi_c(x_m) + \cdots\cdots \right\}$$

termos com 2 ou mais momentos não nulos

Mas $\displaystyle\int \frac{dk_i}{(2\pi)^4}\, e^{ik_i\cdot(x_i+x)} = \delta^{(4)}(x_i+x)$

$$\Rightarrow i\Gamma[\phi_c] = \sum_m \frac{1}{m!}\int dx \left\{ \Gamma^{(m)}_{(0,0\cdots,0)}\, \phi_c^m(x) + \cdots \right\}$$

$$\boxed{\Rightarrow -iV_{eff}(\phi_c) = \sum_m \frac{1}{m!}\,\Gamma^{(m)}_{(0,\cdots,0)}\,\phi_c^m(x)}$$

$$\boxed{V_{eff}(\phi_c) = \sum_m \frac{1}{m!}\, i\,\Gamma^{(m)}_{(0,\cdots,0)}\,\phi_c^m(x)}$$

$i\Gamma^{(2)}_{(0)} = \mu^2$; $i\Gamma^{(4)}_{(0)} = \lambda$ $\left(\Rightarrow \Gamma^{(4)}_{(0)} = -i\lambda\,\nu\right)$

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