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Cosmological solutions in bounded curvature models

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MOTIVATION

- Singularity at the early stage of the universe
  - One of the problems of cosmology in general relativity

- Limited curvature hypothesis *Markov 1982; Markov, Mukhanov 1985; Ginsburg et al. 1988*
  - There is a fundamental scale limiting the curvature invariants

    Analogous to the limitation on the velocity by c in Special Relativity

  - Non-singular cosmology with a finite number of bounded curvature invariants *Mukhanov et al. 1993; Chamseddine, Mukhanov 2016; Yoshida et al. 2017*

MOTIVATION

- Renormalization group method gives the relation between the coupling constant and the field strength.

- Einstein gravity theory may have some corrections from the higher order curvature invariants like $\mathcal{L}_G = \sqrt{-g}\,R \to \sqrt{-g}\,f(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}, \cdots)$

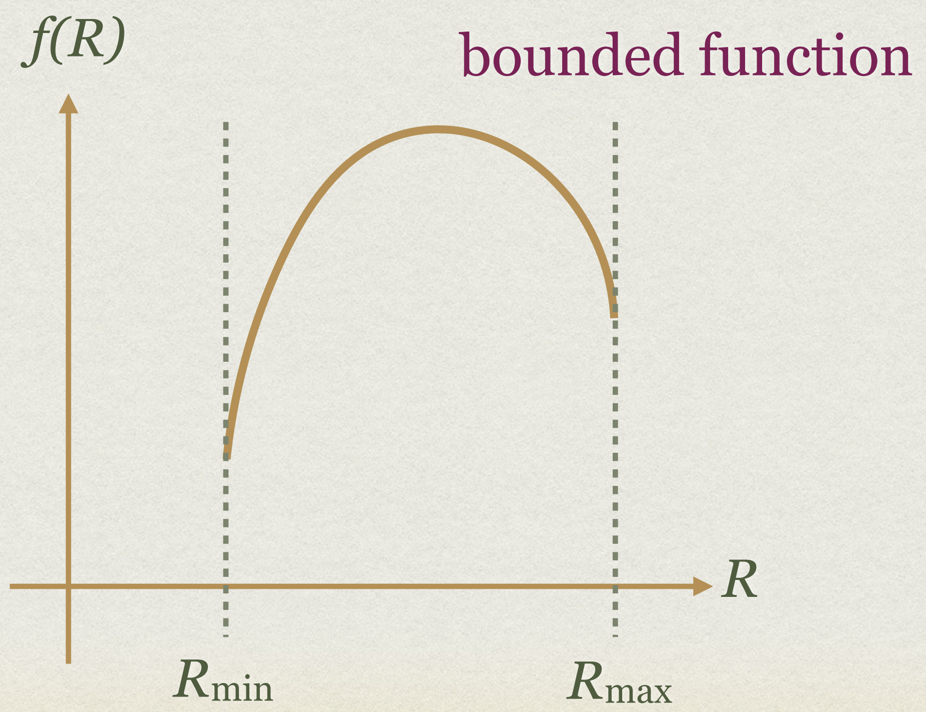
- Our idea: the boundedness of the curvature invariants is dynamically realized if the function *f* is bounded where its derivatives have divergence.

- The simplest case is bounded *f*(*R*)
  (Partial realization of limiting curvature hypothesis)

MOTIVATION

Picking up the feature that the derivatives ($f_R = \partial f/\partial R$, ...) are divergent,

- we discuss cosmological solutions w/o assuming precise functional form of $f(R)$
- we discuss how the boundedness of the curvature affect the existence/behavior of cosmological solutions.



SETUP

- Metric *f(R)* gravity theory ... consider the metric compatible connection

$$S_{\mathrm{G}} = \int d^4x \sqrt{-g} f(R)$$

- FLRW metric ansatz

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]$$

- Matter: vacuum
- Spatially flat background $K=0$

- Simple enough to solve

$$\dot{R} = \frac{1}{3 f_{RR} H} \left[\frac{1}{2} (f_R R - f) - 3 f_R H^2 \right] ,$$

$$\dot{H} = \frac{R}{6} - 2H^2 ,$$

- Analysis in 2 dim phase space (R, H)

STATIONARY POINTS

$$\dot{R} = 0 \,, \quad \dot{H} = 0 \,.$$

$$H_{\pm}(R) = \pm\sqrt{\frac{R}{12}}$$

$$\dot{R} = \pm\frac{f_R R - 2f}{2\sqrt{3}R^{1/2} f_{RR}} = 0$$

Two Possibilities

$f_R R - 2f = 0$

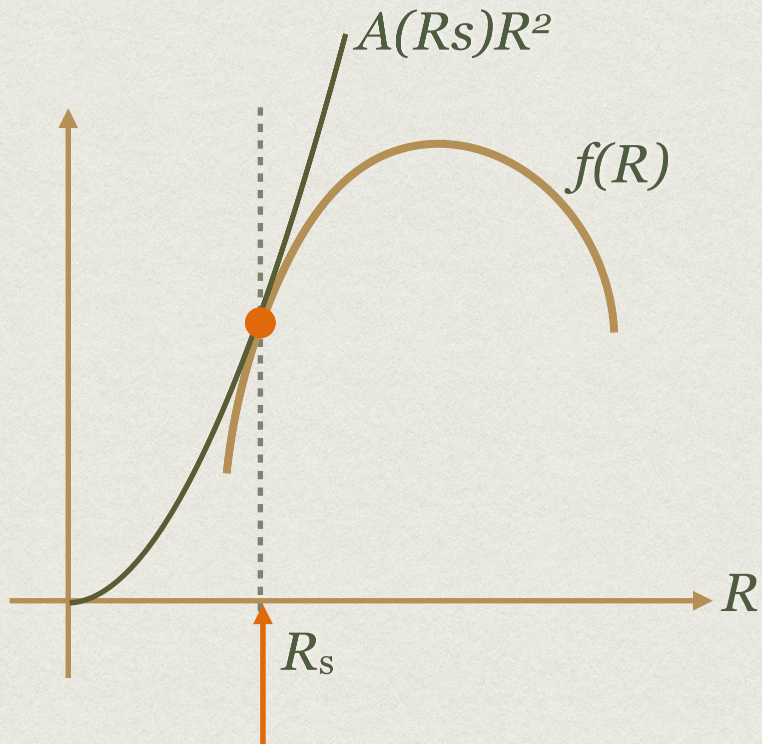
- Regular-stationary points
  Realized for curvatures in (R_{min} , R_{max})

$f_{RR} \to -\infty$

- Extremal-stationary points
  Realized at the maximum/minimum curvatures

REGULAR-STATIONARY POINTS

- What's the meaning of the condition $f_R R - 2f = 0$
  - <u>Find the quadratic curve(s) tangential to *f(R)*</u>



R, at which regular-stationary points are found

NON-STATIONARY CHARACTERISTIC POINTS

- Near R-axis, i.e., $H \sim 0$

$$\dot{R} = \frac{f_R R - f}{6 f_{RR} H}$$

Divergent except for $f_R R - f = 0$

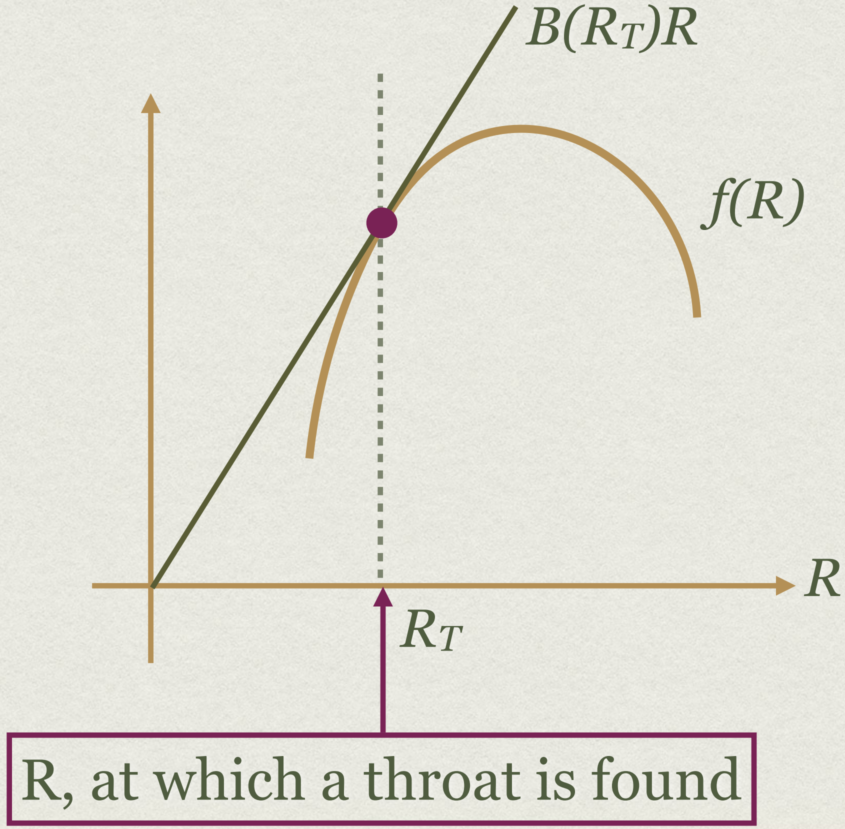
- Almost all the points on H=0 are not allowed
- At $f_R R - f = 0$,
  - Flows pass through the point in a finite time.
  - A solution exists for any initial condition near the throat. Uniqueness condition is broken. (w/o Lipschitz continuity)

Throats connecting the $H<0$ region and $H>0$ the region

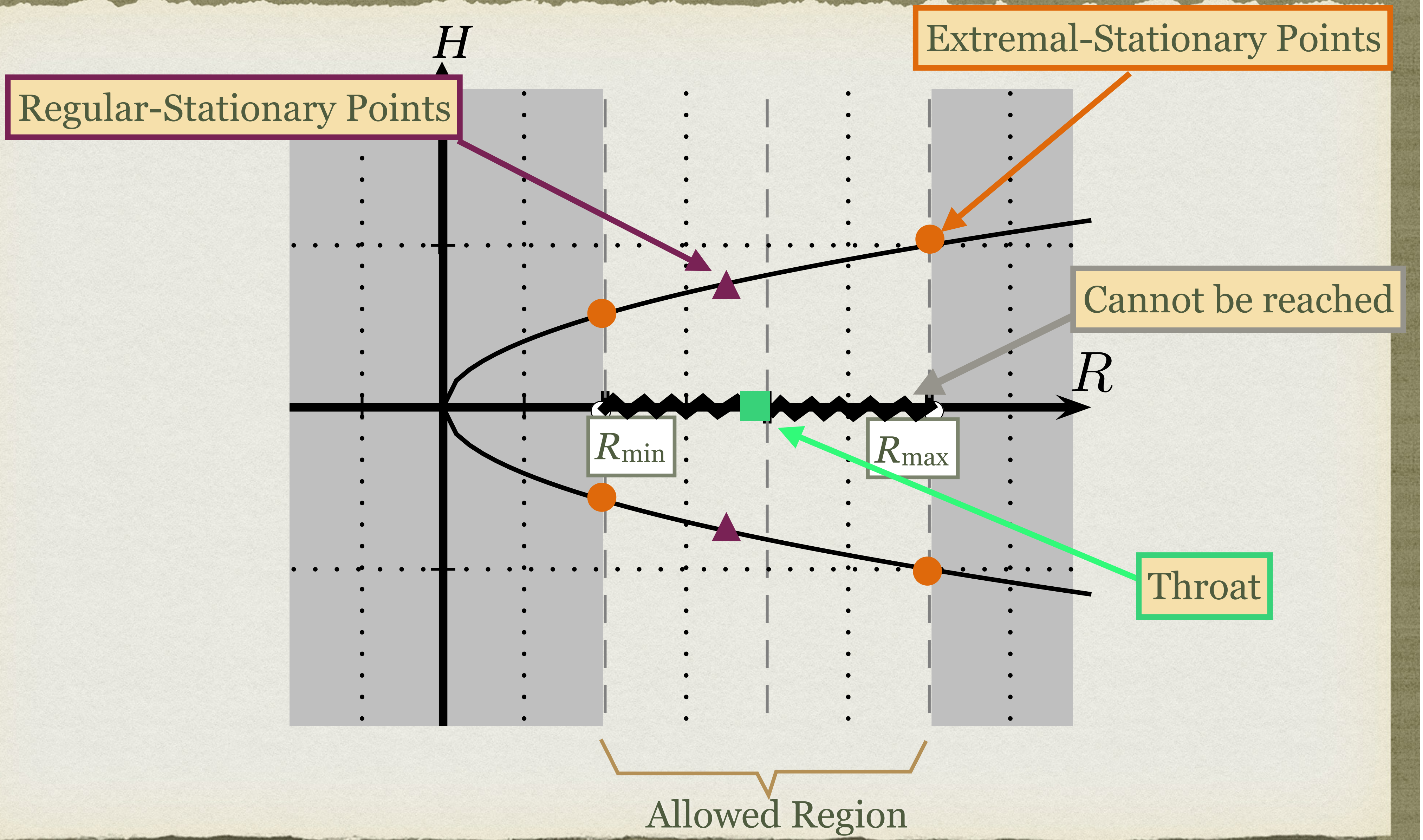
  - Such the point is unique for a convex function *f(R)*

NON-STATIONARY CHARACTERISTIC POINTS

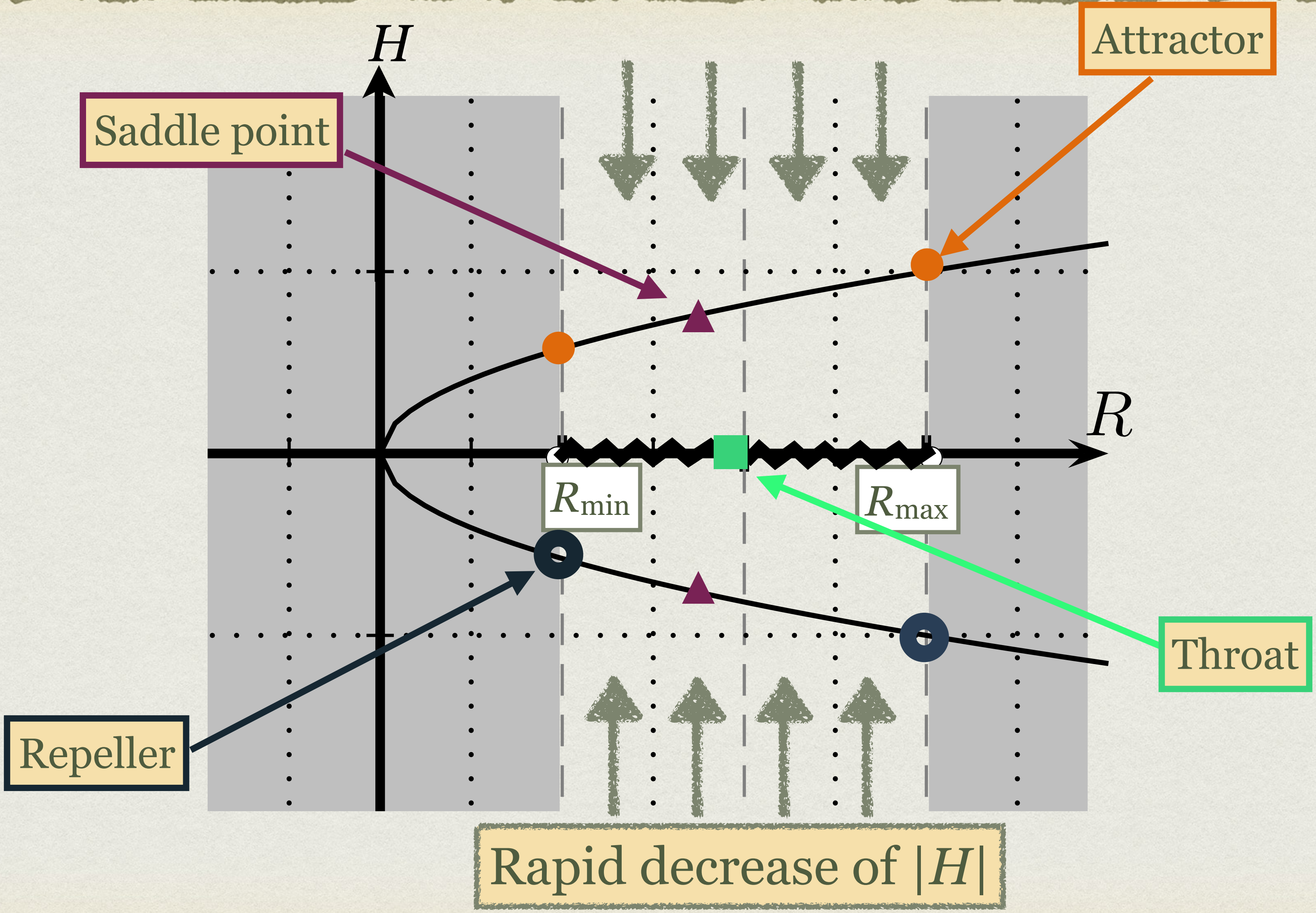
- What's the meaning of the condition $f_R R - f = 0$
    - Find the linear function(s) tangential to *f(R)*



GLOBAL STRUCTURE



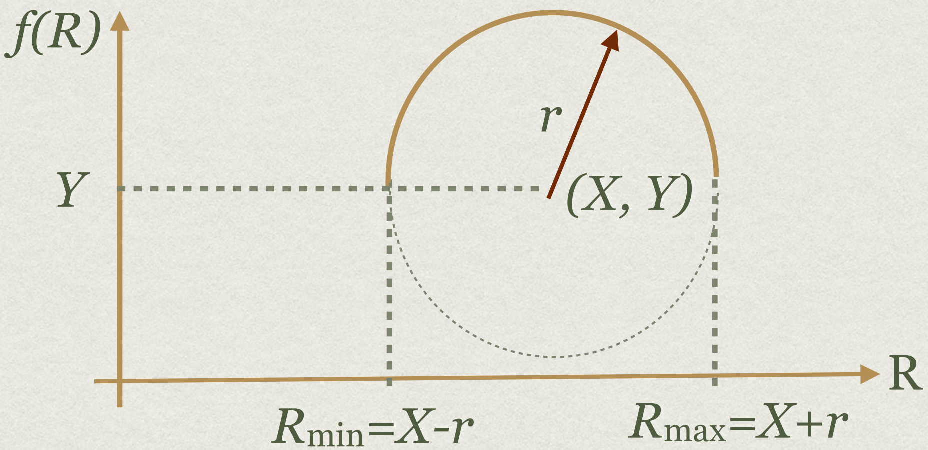
STABILITY AND ASYMPTOTIC BEHAVIORS



HEMI-CIRCULAR MODEL

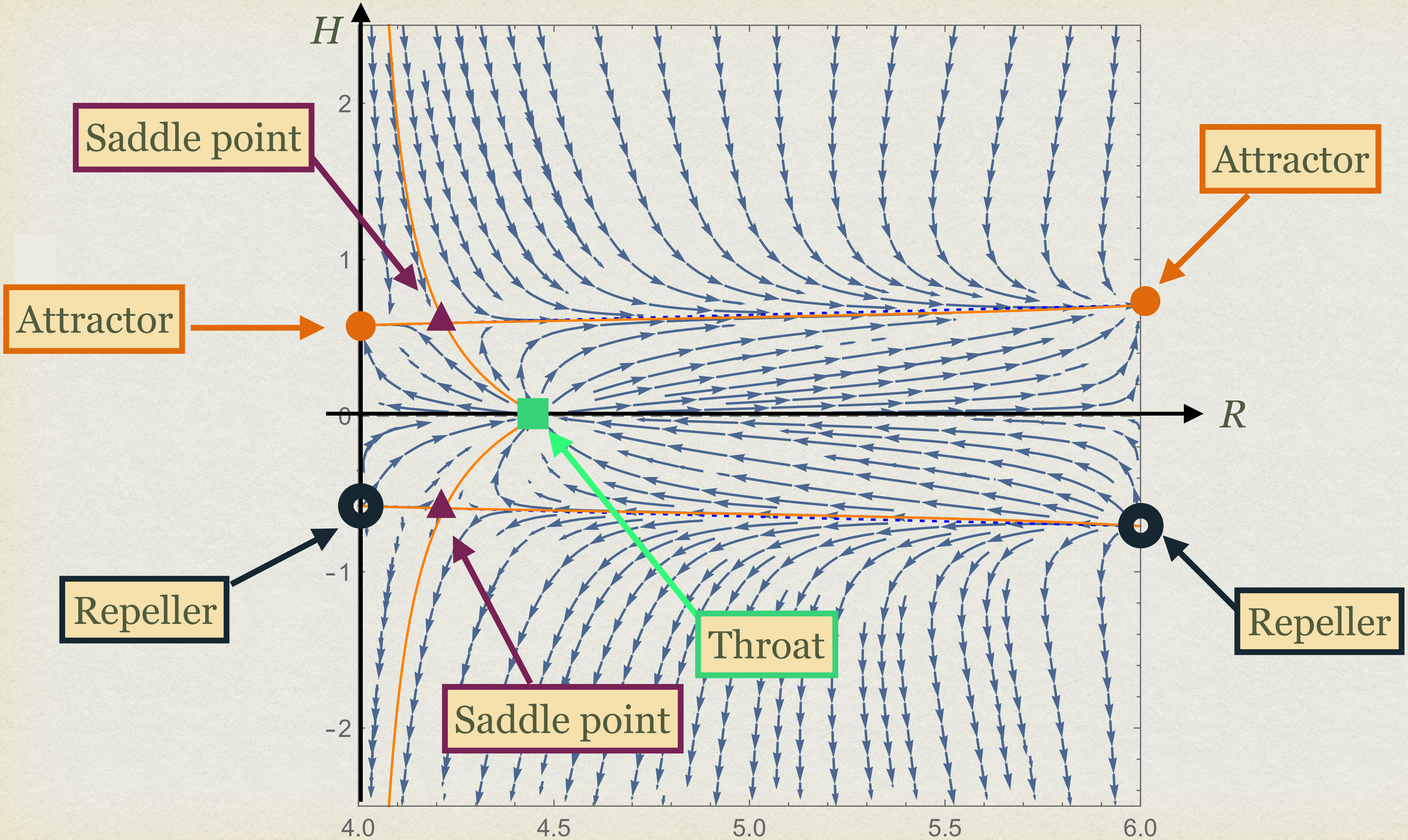
- Upper hemi-circle

$$f_{+}(R) = Y + \sqrt{r^2 - (R - X)^2} \qquad \text{defined in} \qquad X - r \leq R \leq X + r$$



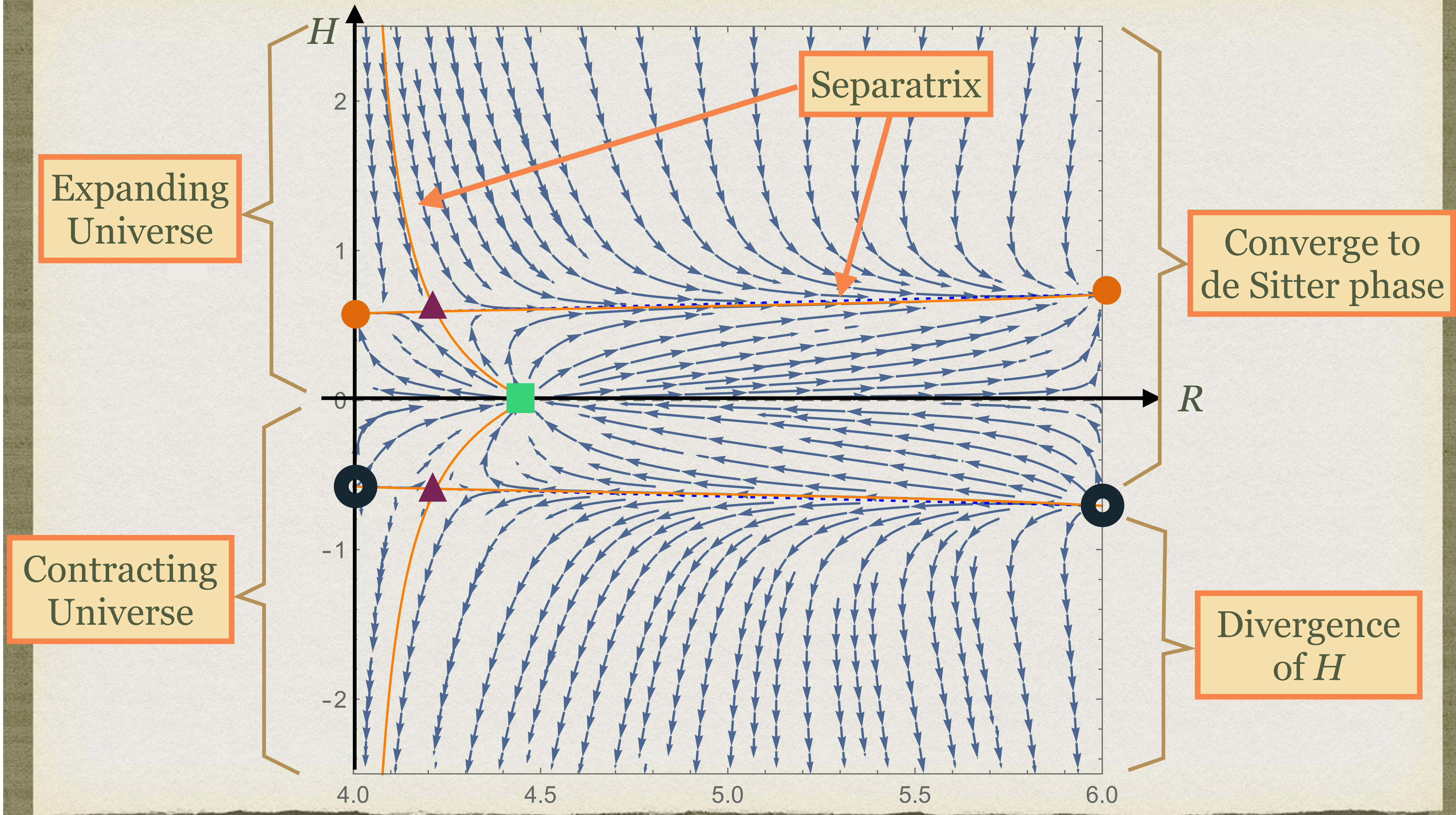
PHASE SPACE

$(X, Y)=(5, 2)$



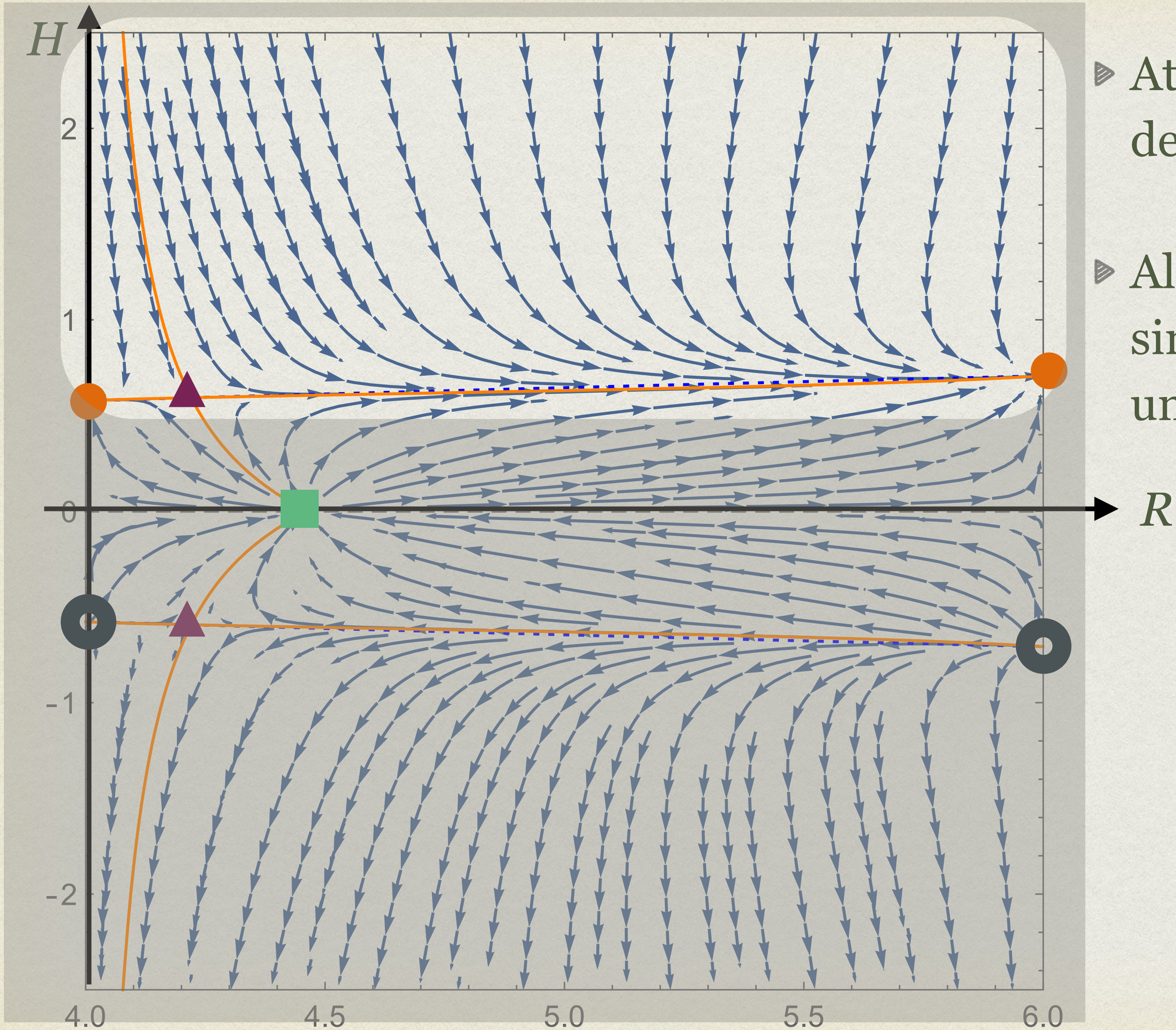
PHASE SPACE

(X, Y)=(5, 2)



INTERPRETATION

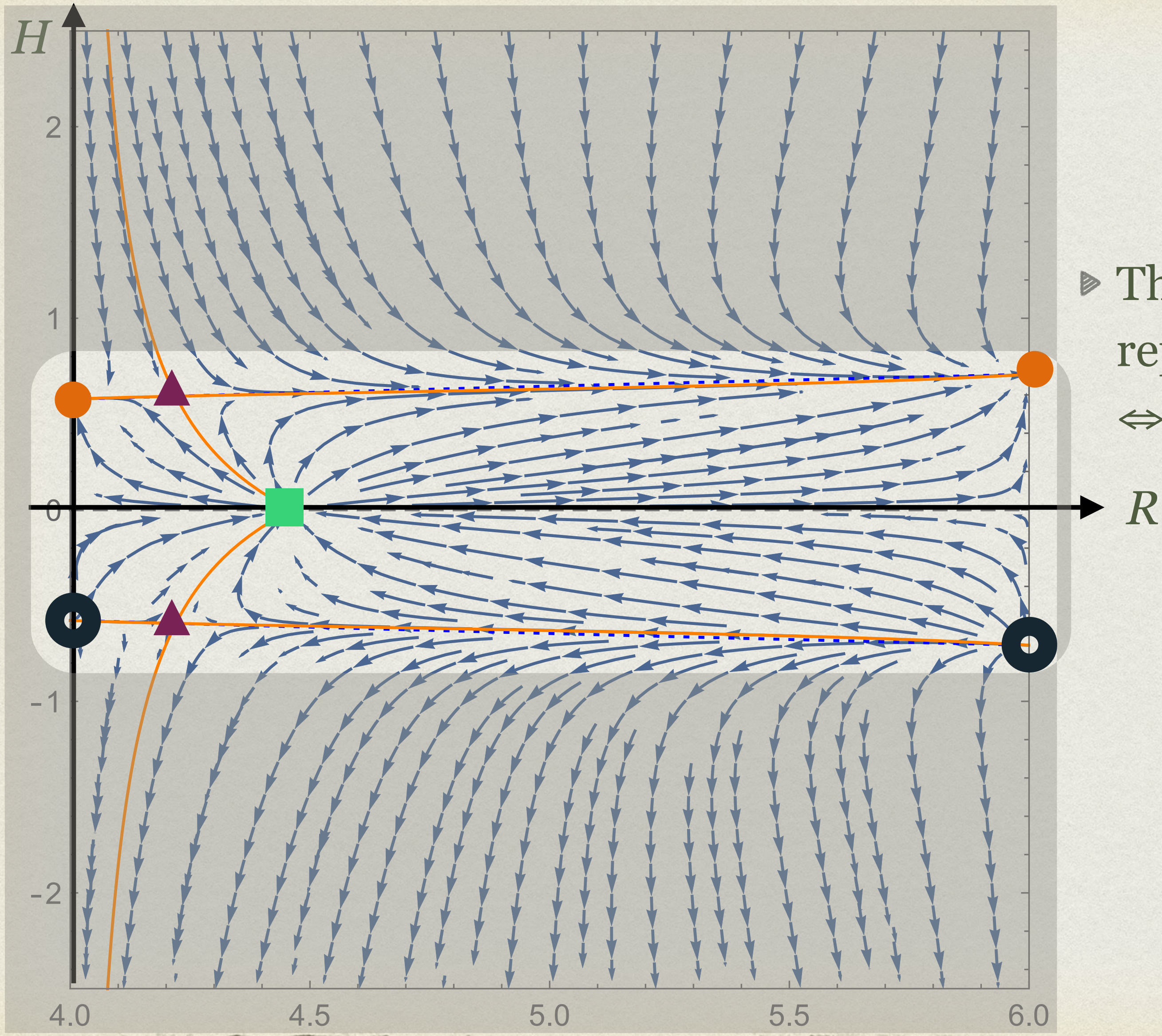
(X, Y)=(5, 2)



- At the attractors/repellers, de Sitter spacetime is realized
- Almost constant *H* phase has similar behavior to inflationary universe

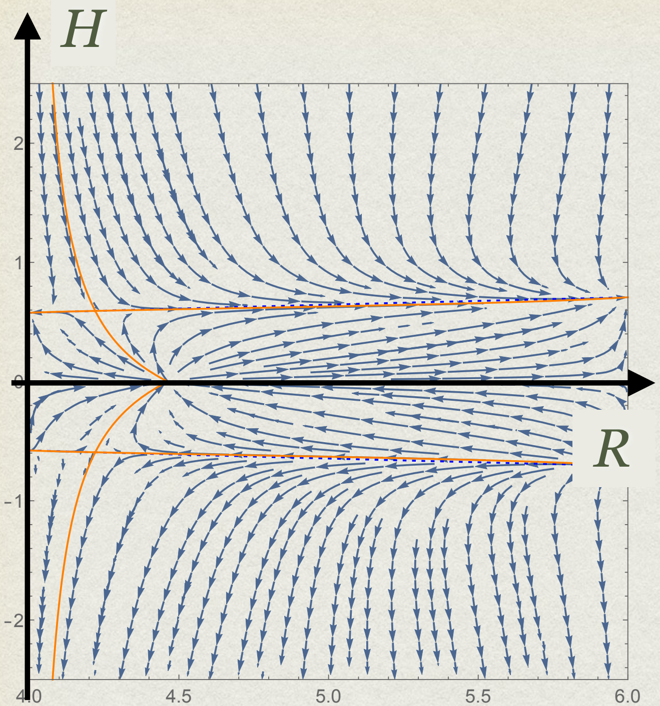
INTERPRETATION

(X, Y)=(5, 2)

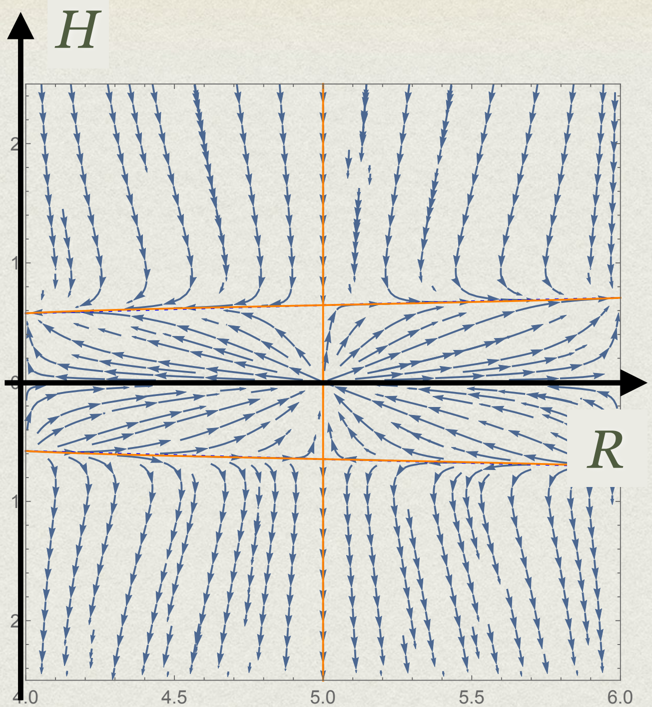


- The evolution from a de Sitter repeller to a de Sitter attractor ⇔ Bouncing solutions

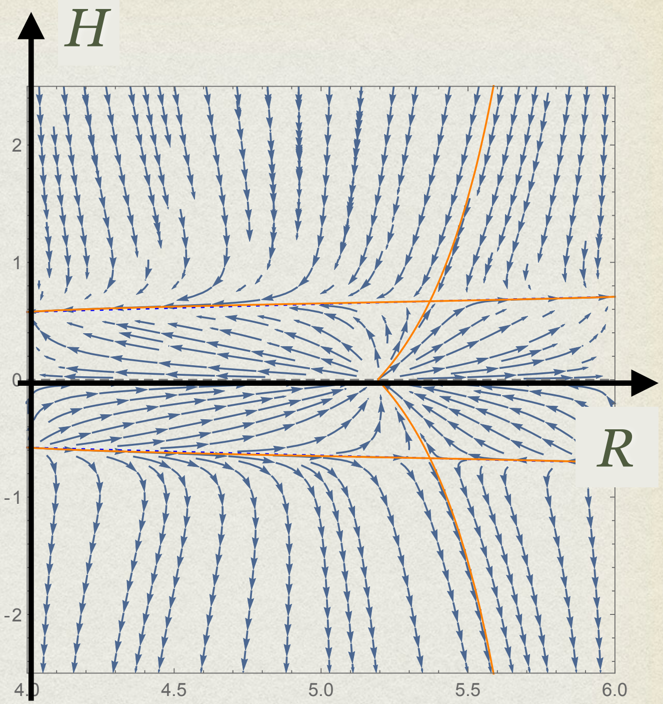
CHANGE OF PARAMETER



(X, Y)=(5, 2)



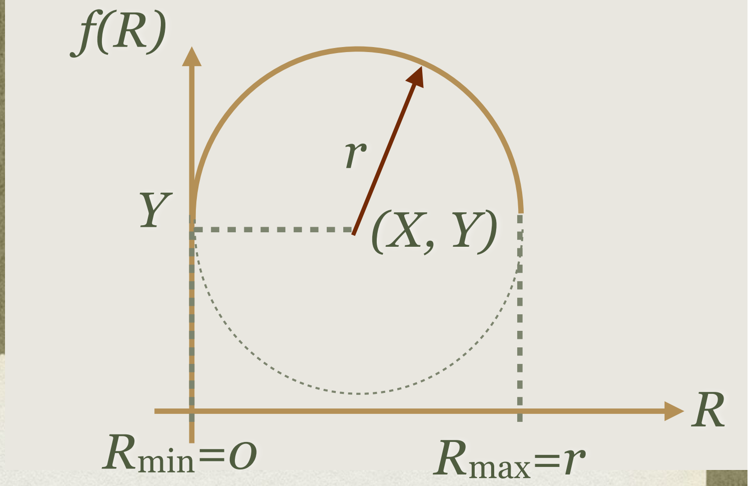
(X, Y)=(5, -1)



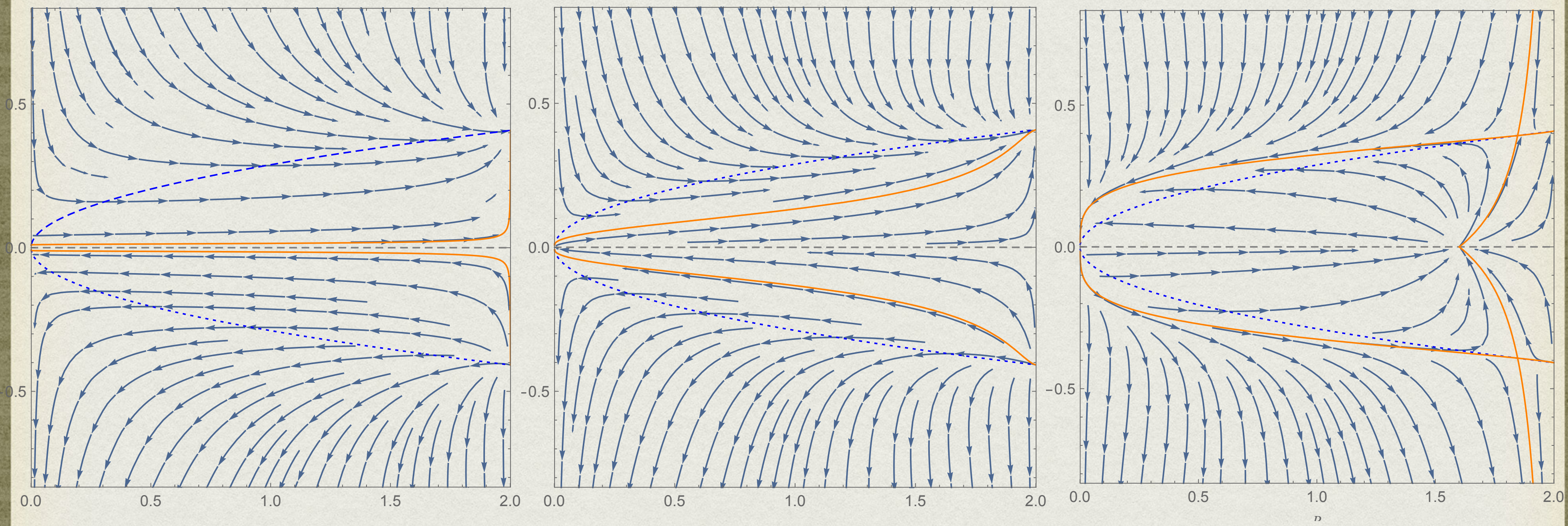
(X, Y)=(5, -2)

- The exact position of characteristic points changes
- Their existence and stability never change for $R_{\rm min}>0$ models

R_{MIN} = 0 CASES

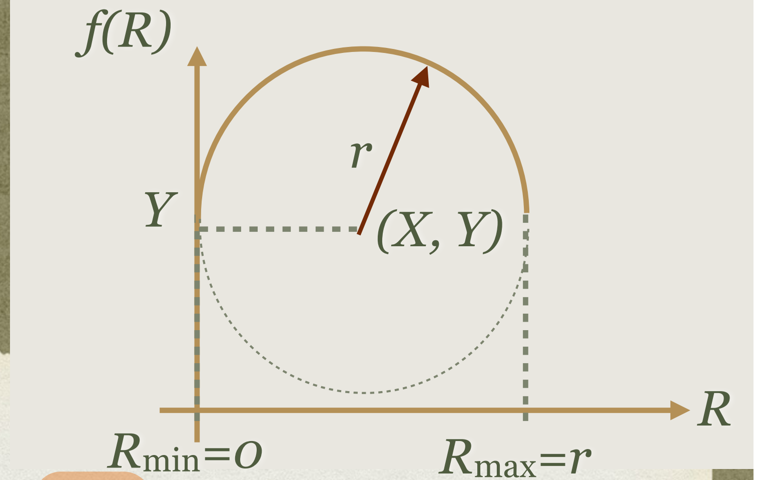


- Phase space

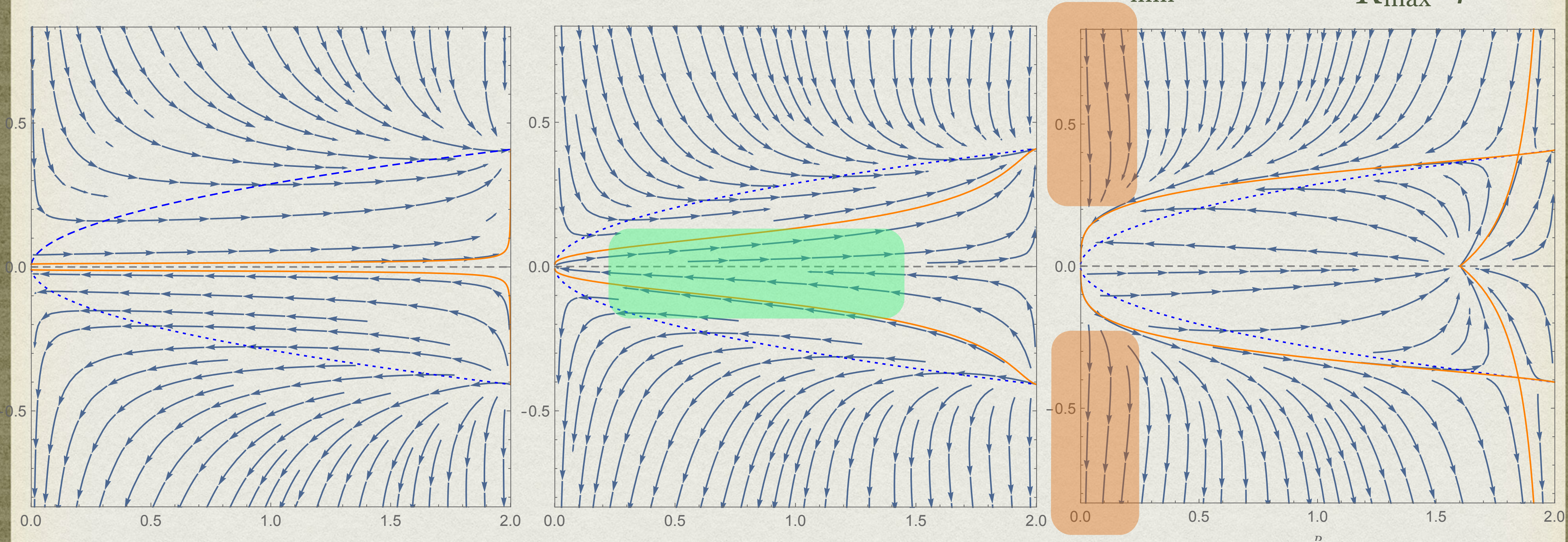


Y>0 | *Y*=0 | *Y*<0

$R_{MIN} = 0$ CASES



- Phase space



Y>0 *Y=0* *Y<0*

- There are two cases which can reach Minkowski spacetime

Y=0 and R>>H²

Y<0 and R<<H²

SUMMARY

- We motivated bounded curvature models with divergence in *the derivatives of f(R)* at the boundaries by referring to the renormalization of the gravitational constant.

- *Flat-FLRW* solutions in *convex and bounded f(R)* theories.
  Stability of stationary points, the structure of phase space (R, H), ...

- Explicit example: *f(R) in hemi-circular form*.

SUMMARY

- We have found two typical types of evolution:
  One experiences
  (1) Rapid decrease of H,
  (2) Almost constant H phase,
  (3) Convergence into de Sitter attractor;
  the other experiences
  (1) Contraction phase,
  (2) Expanding phase after the passage through the throat
  (3) Convergence into de Sitter attractor.

- The difference of $R_{\mathrm{min}}=0$ cases from $R_{\mathrm{min}}>0$ cases is
  (1) Phase space is more sensitive to the choice of *f(R) (i.e., of parameters)*
  (2) The spacetime converges into Minkowski.

FUTURE PLAN

- We discussed flat FLRW case.
  → How about negative *R* region. Inclusion of the spatial curvature.

  Similar treatment

- We discussed vacuum cases.
  → How about inclusion of matter.
  We are now discussing. The results seem positive.

- We discussed only the evolution of background spacetime.
  → How about the perturbative stability.
  There is a correspondence between *f(R)* and Brans-Dicke theories at regular curvatures, away from the maximum/minimum curvatures.

- We discussed cosmological solutions.
  → How about the realization of GR in our solar system.
  Screening mechanism of the fifth force at the minimum curvature?