(Toward) new aspects of massive gravity

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Massive Gravity

- General relativity + **non-zero** graviton's mass

$$S = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\Big[R - \text{Mass term}\Big]$$

- Is it possible to construct such a massive general relativity?
- What is the consequence of introducing mass?
- Could it be responsible for the current accelerated expansion of the universe?
- Other interesting features…?

Fierz-Pauli Theory

- Fierz-Pauli theory (Fierz, Pauli, 1939)

$$S = M_{\rm Pl}^2 \int d^4x \left[\boxed{-\frac{1}{2} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta}} \boxed{- \frac{1}{4} m^2 (h_{\mu\nu}h^{\mu\nu} - h^2)} \right]$$

Linearized Einstein-Hilbert term

Only allowed mass term which does not have ghost at linear order

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} = -\frac{1}{2}(\Box h_{\mu\nu} - \partial_\mu \partial_\alpha h^\alpha_\nu - \partial_\nu \partial_\alpha h^\alpha_\mu + \partial_\mu \partial_\nu h^\alpha_\alpha - \eta_{\mu\nu} \Box h^\alpha_\alpha + \eta_{\mu\nu} \partial_\alpha \partial_\beta h^\alpha_\beta)$$

(1) Lorentz invariant theory

(2) Gauge invariance is broken due to the mass term

(3) No ghost (5 DOF = 2 tensor + 2 vector + 1 scalar)

(4) **Simple nonlinear extension contains ghost at nonlinear level**

(Boulware-Deser ghost, 6th DOF) (Boulware, Deser, 1971)

dRGT Massive Gravity

- dRGT massive gravity (**d**e **R**ham, **G**abadadze, **T**olley 2011)

$$S_{MG} = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}\left(\mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4\right)\right] + S_m[g_{\mu\nu},\psi]$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \sqrt{\delta^\mu{}_\nu - H^\mu{}_\nu} = \delta^\mu{}_\nu - \left(\sqrt{g^{-1}\eta}\right)^\mu{}_\nu$$

$$H_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$$
$$\sqrt{X^\mu{}_\alpha}\sqrt{X^\alpha{}_\nu} = X^\mu{}_\nu$$

$$\mathcal{U}_2 = 2\varepsilon_{\mu\alpha\rho\sigma}\varepsilon^{\nu\beta\rho\sigma}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta = 4\left([\mathcal{K}^2] - [\mathcal{K}]^2\right)$$
$$\mathcal{U}_3 = \varepsilon_{\mu\alpha\gamma\rho}\varepsilon^{\nu\beta\delta\rho}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta\mathcal{K}^\gamma{}_\delta = -[\mathcal{K}]^3 + 3[\mathcal{K}][\mathcal{K}^2] - 2[\mathcal{K}^3]$$
$$\mathcal{U}_4 = \varepsilon_{\mu\alpha\gamma\rho}\varepsilon^{\nu\beta\delta\sigma}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta\mathcal{K}^\gamma{}_\delta\mathcal{K}^\rho{}_\sigma = -[\mathcal{K}]^4 + 6[\mathcal{K}]^2[\mathcal{K}^2] - 3[\mathcal{K}^2]^2 - 8[\mathcal{K}][\mathcal{K}^3] + 6[\mathcal{K}^4]$$

- Expanding the square root in the potential term

$$\mathcal{U}_2 = \boxed{[H^2] - [H]^2} \boxed{- \frac{1}{2}([H][H^2] - [H^3]) + \mathcal{O}(H^4)}$$

Fierz-Pauli mass term **Infinite nonlinear corrections to eliminate BD ghost**

No BD ghost at full order (5 DOF) (Hassan, Rosen, 2011)

Non-canonical Kinetic Term

(**RK** & Yamauchi, 2013)

- dRGT mass term is uniquely determined
- Is dRGT theory a unique theory describing massive graviton without introducing other fields ?

$$S_{MG} = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}\left(\mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4\right)\right]\boxed{+ S_{int}} + S_m[g_{\mu\nu}, \psi],$$

New Kinetic terms???

- Candidates for derivative interactions

$$\cancel{\begin{aligned}\mathcal{L}_{int} \supset & M_{\rm Pl}^2\sqrt{-g}\, g_{..} H_{..} R^{....}, M_{\rm Pl}^2\sqrt{-g}\, H_{..} H_{..} R^{....}, \cdots \\ & M_{\rm Pl}^2\sqrt{-g}\, \nabla_. \nabla_. H^{.}{}_{.} H^{..}, M_{\rm Pl}^2\sqrt{-g}\, \nabla_. \nabla_. H^{.}{}_{.} H^{.}{}_{.} H^{..}, \cdots\end{aligned}}$$

- **No-go theorem** - no derivative interaction cannot be introduced in dRGT theory due to the appearance of the BD ghost. (RK & Yamauchi 2013, de Rham et al. 2013)

- A Staring point of proof is FP theory in both [RK, Yamauchi 2013] & [de Rham et al. 2013]

General Spin-2 Theory

- Our Lagrangian

$$S = \int \mathrm{d}^4x \Big(-\mathcal{K}^{\alpha\beta|\mu\nu\rho\sigma} h_{\mu\nu,\alpha} h_{\rho\sigma,\beta} - \mathcal{M}^{\mu\nu\rho\sigma} h_{\mu\nu} h_{\rho\sigma} \Big)$$

$$\begin{aligned} \mathcal{K}^{\alpha\beta|\mu\nu\rho\sigma} &= \kappa_1 \eta^{\alpha\beta}\eta^{\mu\rho}\eta^{\nu\sigma} + \kappa_2 \eta^{\mu\alpha}\eta^{\rho\beta}\eta^{\nu\sigma} + \kappa_3 \eta^{\alpha\mu}\eta^{\nu\beta}\eta^{\rho\sigma} + \kappa_4 \eta^{\alpha\beta}\eta^{\mu\nu}\eta^{\rho\sigma} \,, \\ \mathcal{M}^{\mu\nu\rho\sigma} &= \mu_1 \eta^{\mu\rho}\eta^{\nu\sigma} + \mu_2 \eta^{\mu\nu}\eta^{\rho\sigma} \,, \end{aligned}$$

$\kappa_i, \mu_{1,2}$: (constant) free parameters

General relativity

Linearized Einstein-Hilbert term $\quad \kappa_2 = -\kappa_3 = 2\kappa_4 = -2\kappa_1 \quad \mu_1 = \mu_2 = 0$

Gauge symmetry $\quad h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

Fierz-Pauli theory

Kinetic term = Linearized Einstein-Hilbert term

Gauge invariance is broken by the FP mass term $\mu_1 = -\mu_2$

Finding Theory

- SVT decomposition

$$h_{00} = h^{00} = -2\alpha\,, \qquad h_{0i} = -h^{0i} = \beta_{,i} + B_i \quad (B^i{}_{,i} = 0)$$

$$h_{ij} = h^{ij} = 2\mathcal{R}\delta_{ij} + 2\mathcal{E}_{,ij} + F_{i\,,j} + F_{j\,,i} + 2H_{ij} \quad (F^i{}_{,i} = 0\,, \quad H^i{}_i = H^{ij}{}_{,j} = 0)$$

$$\Rightarrow \quad S = S^S[\alpha\,,\beta\,,\mathcal{R}\,,\mathcal{E}] + S^V[B_i\,,F_i] + S^T[H_{ij}]$$

- Tensor sector

$$S^T[H_{ij}] = 4\int \mathrm{d}t\,\mathrm{d}^3k\Big[\kappa_1\dot{H}_{ij}^2 - (\kappa_1 k^2 + \mu_1)H_{ij}^2\Big]$$

  - Ghost-free condition $\kappa_1 > 0$
  - Tachyonic instability is absent when $\mu_1 \geq 0$
  - Degrees of freedom = 2

VECTOR SECTOR

- B_i and F_i has 4 DOFs
- We need to eliminate one of them

  (Otherwise, ghost or gradient instabilities appears in B or F.)

$$S^V[B_i\,,F_i] = \int \mathrm{d}t\,\mathrm{d}^3k\Big[-(2\kappa_1+\kappa_2)\dot{B}_i^2 + 2\kappa_1\dot{F}_i^2 + 2\kappa_2 k B_i\dot{F}_i + 2\left(\kappa_1 k^2+\mu_1\right)B_i^2 - \left(k^2(2\kappa_1+\kappa_2)-2\mu_1\right)F_i^2\Big]$$

$$\Big\downarrow\ 2\kappa_1+\kappa_2=0$$

$$S^V = \int \mathrm{d}t\,\mathrm{d}^3k\Big[2\kappa_1\dot{F}_i^2 - 4\kappa_1 k B_i\dot{F}_i + 2\left(\kappa_1 k^2-\mu_1\right)B_i^2 - 2\mu_1 F_i^2\Big]$$

- Canonical momenta

$$\pi_{B_i} = 0\,, \xrightarrow{\text{Primary constraints}} \mathcal{C}_1^{B_i} = \pi_{B_i} = 0$$

$$\pi_{F_i} = 4\kappa_1(\dot{F}_i - kB_i)$$

- Secondary constraints

$$\mathcal{C}_2^{B_i} \equiv \dot{\mathcal{C}}_1^{B_i} = \{\mathcal{C}_1^{B_i}\,, H_T\} = \{\mathcal{C}_1^{B_i}\,, H\} = k\pi_{F_i} + 4\mu_1 B_i \approx 0$$

- Time-evolution of the secondary constraints

$$\dot{\mathcal{C}}_2^{B_i} = \{\mathcal{C}_2^{B_i}\,, H_T\} = \{\mathcal{C}_2^{B_i}\,, H\} + \lambda_{B_j} \underbrace{\{\mathcal{C}_2^{B_i}\,, \mathcal{C}_1^{B_j}\}}_{= 4\mu_1\,\delta_{ij}} \approx 0\,,$$

Case V1 : $\mu_1 \neq 0$

$$\text{vector DOFs} = \frac{4 \times 2 - 4\,(2 \text{ primary } \& \text{ 2 secondary})}{2} = 2$$

Case V2 : $\mu_1 = 0$

$$\text{vector DOFs} = \frac{4 \times 2 - 4\,(2 \text{ primary } \& \text{ 2 secondary}) \times 2\,(\text{first-class})}{2} = 0$$

Scalar Sector

- Classification based on the Hamiltonian analysis

Case	DOF	Conditions	Free parameters	Comments
SI & V1	$3 = 2 + 0 + 1$	$\mu_1 = 0$	$\kappa_3, \kappa_4, \mu_2$	New theories
SIIa & V1	$2 = 2 + 0 + 0$	"Condition 3" & $\mu_1 = \mu_2 = 0$	κ_3	General relativity is included
SIIb & V1	$2 = 2 + 0 + 0$	"Condition 3" & $\mu_1 = 0$ & $\mu_2 \neq 0$	κ_3, μ_2	New theories
SIIc & V2	$5 = 2 + 2 + 1$	"Condition 3 & 4" & $\mu_1 \neq 0$	κ_3, μ_1	Fierz-Pauli is included

$$\text{[Condition 2]} \ : \ 2\kappa_1 + \kappa_2 = 0$$

$$\text{[Condition 3]} \ : \ 4\kappa_1^2 - 4\kappa_1\kappa_3 + 8\kappa_1\kappa_4 + 3\kappa_3^2 = 0$$

$$\text{[Condition 4]} \ : \ \mu_2 = -\frac{\mu_1}{4\kappa_1^2}(4\kappa_1^2 - 6\kappa_1\kappa_3 + 3\kappa_3^2)$$

- All theories satisfies "Condition 2" (for healthy vector modes)
- I, IIa, & IIb has gauge symmetries (containing first-class constraints)
- Massless limit $\mu_1 \to 0$ of the case IIc reduces to the case IIa

  (# of DOF is different.)

Partially massless case (case I)

(DOF=3 case)

$$\text{scalar DOFs} = \frac{4\times 2 - 3\,(1\ \text{primary}\ \&\ 1\ \text{secondary}\ \&\ 1\ \text{tertiary})\times 2\,(\text{first class})}{2} = 1$$

- Gauge symmetry

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \qquad \text{with} \quad \partial^\mu \xi_\mu = 0$$

(Transverse diffeomorphisms [J. J. Van der Bij. et.al, 1982])

- Lagrangian with gauge invariant variables

$$\kappa_1 = 1/8, \quad \kappa_3 = 1/4, \quad \kappa_4 = 1/8$$

$$\mathcal{L}_{\mathrm{I}}^{S} = \boxed{-3\dot{\mathcal{R}}^2 + k^2\mathcal{R}^2 + 2k^2\tilde{\alpha}\mathcal{R}} + \boxed{\dot{\tilde{\mathcal{E}}}^2 - (k^2 + 4\mu_2)\tilde{\mathcal{E}}^2}$$

Non-dynamical parts **Dynamical parts**

$$\tilde{\mathcal{E}} = \mathrm{Tr}\, h_{\mu\nu}|_{\text{scalar}}$$

Massless case (case IIa)

(Massless, DOF=2 case)

$$\text{scalar DOFs} = \frac{4 \times 2 - 4\,(2\ \text{primary}\ \&\ 2\ \text{secondary}) \times 2\,(\text{first-class})}{2} = 0$$

- Gauge symmetry

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + b\, \partial^\rho \xi_\rho \eta_{\mu\nu}$$

$$b = -\frac{2\kappa_1 - \kappa_3}{2(\kappa_1 - \kappa_3)}$$

General relativity

Linearlized Einstein-Hilbert term $\quad \kappa_2 = -\kappa_3 = 2\kappa_4 = -2\kappa_1$

Gauge symmetry $\quad h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

***b = 0* for Einstein-Hilbert term**

Massless case (case IIb)

(Massless, DOF=2 case)

$$\text{scalar DOFs} = \frac{4\times 2 - 3\,(1\text{ primary \& 1 secondary \& 1 tertiary})\times 2\,(\text{first-class}) - 2\,(1\text{ primary \& 1 secondary})}{2} = 0$$

- Gauge symmetry

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \qquad \text{with} \quad \partial^\mu \xi_\mu = 0$$

(Transverse diffeomorphisms [J. J. Van der Bij. et.al, 1982])

- Lagrangian with gauge invariant variables

$\kappa_1 = 1/8, \quad \kappa_3 = 1/4$

$$\mathcal{L}^S_{\mathrm{IIa}} = \boxed{-3\dot{\mathcal{R}}^2 + k^2\mathcal{R}^2 + 2k^2\tilde{\alpha}\mathcal{R}}\boxed{-4\mu_2\tilde{\mathcal{E}}^2}$$

Non-dynamical **Non-dynamical**

$$\tilde{\mathcal{E}} = \mathrm{Tr}\, h_{\mu\nu}|_{\mathrm{scalar}}$$

Field Redefinition

- Possible field redefinition of $h_{\mu\nu}$

$$\tilde{h}_{\mu\nu} = h_{\mu\nu} + \Gamma \, \mathrm{Tr}[h_{\alpha\beta}] \, \eta_{\mu\nu}$$

- One can map the new theory to GR & FP theory

Case IIa $\longleftrightarrow$ Linearized GR

$$\Gamma = -\frac{2\kappa_1 - \kappa_3}{4\kappa_1}$$

Case IIc $\longleftrightarrow$ Fierz-Pauli theory

These theories can be mapped from GR and Fierz-Pauli theories

(In the absence of the coupling to extra field, these are the same theories)

Summary

- **New** kinetic and mass interactions for spin-2 theories
  - 4 independent classes
    - Case I : DOF = 3 [2 tensor + 1 scalar]
    - Case IIa : DOF = 2 [2 tensor] (include GR)
    - Case IIb : DOF = 2 [2 tensor]
    - Case IIc : DOF = 5 [2 tensor + 2 vector + 1 scalar] (include FP)
  - Class IIa and IIc can be mapped with field redefinition from linearized general relativity and Fierz-Pauli theory
- Matter coupling might be a problem for case IIc…

$$\mathcal{L}^{(\mathrm{DL})} = \mathcal{L}^{(\mathrm{DL})}_{\mathrm{tensor}}[\tilde{h}] - \frac{6c_1}{\kappa_1}(\partial_\mu\pi)^2 + \frac{1}{M}\left[\tilde{h}_{\mu\nu}T^{\mu\nu} - \frac{c_1}{\kappa_1-\kappa_3}\pi T\right] + \frac{b}{\Lambda_3^3}\Box\pi T$$

Summary

- No-go theorem ← true ?
  - The starting point of the previous work was Fierz-Pauli theory
- Nonlinear counterparts of case Iic

$$S = \int \mathrm{d}^4x\sqrt{-g}\,\Big[a_1 R + a_2 hR + a_3 h_{\mu\nu}R^{\mu\nu} + \mathfrak{K}^{\alpha\beta|\mu\nu\rho\sigma}\nabla_\alpha h_{\mu\nu}\nabla_\beta h_{\rho\sigma} + \mathfrak{M}^{\mu\nu\rho\sigma}h_{\mu\nu}h_{\rho\sigma} + \mathcal{O}(h^2R, h^3)\Big]$$

$$\begin{aligned} h_{\mu\nu} &= g_{\mu\nu} - \eta_{\mu\nu} \\ \mathfrak{K}^{\alpha\beta|\mu\nu\rho\sigma} &= b_1 g^{\alpha\beta}g^{\mu\rho}g^{\nu\sigma} + b_2 g^{\mu\alpha}g^{\rho\beta}g^{\nu\sigma} + b_3 g^{\alpha\mu}g^{\nu\beta}g^{\rho\sigma} + b_4 g^{\alpha\beta}g^{\mu\nu}g^{\rho\sigma}\,, \\ \mathfrak{M}^{\mu\nu\rho\sigma} &= \mu_1 g^{\mu\rho}g^{\nu\sigma} + \mu_2 g^{\mu\nu}g^{\rho\sigma}\,, \end{aligned}$$

$$\kappa_3 = b_3 - a_1 - a_2 - \frac{1}{2}a_3\,, \qquad \kappa_4 = b_4 + \frac{1}{2}a_1 + a_2\,.$$

$$\kappa_1 = b_1 - \frac{1}{2}a_1 + \frac{1}{2}a_3\,, \qquad \kappa_2 = b_2 + a_1 - a_3$$

- Does nonlinear theories contain ghost or not ?