Why do we still consider dynamical models of dark energy ?

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$c = \hbar = M_G^2 = 1/(8\pi G) = 1$

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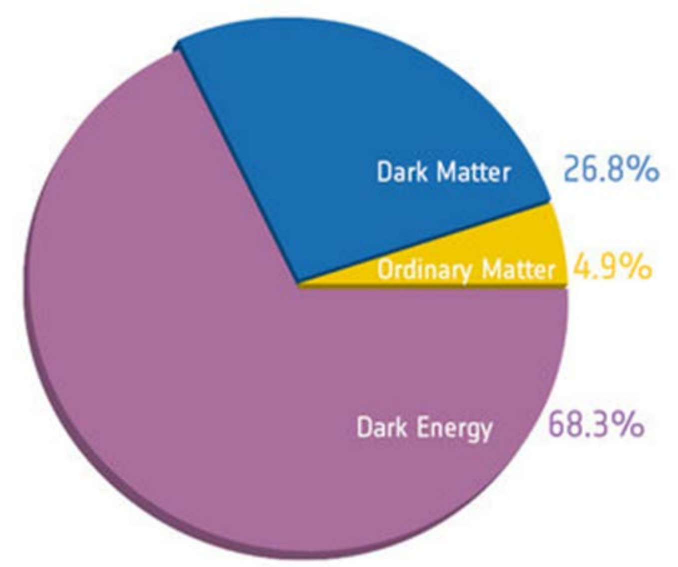
Introduction

Dark energy

Recent observations imply that a component with negative pressure (dark energy) dominates the energy density of the universe.



PLANCK team

w_0 is close to -1 !!
cosmological constant ???

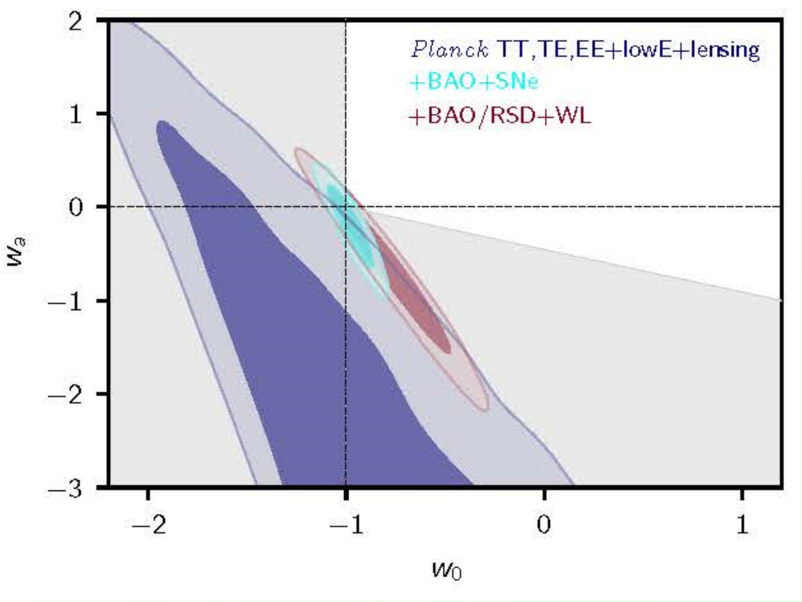
ns to the base-ΛCDM model for combinations of *Planck* power spectra, *Planck* lensing,
ɔec likelihood are given in Table A.2). Note that we quote 95 % limits here.

		TT, TE, EE+lowE	TT, TE, EE+lowE+lensing	TT, TE, EE+lowE+lensing+BAO
[illegible]	[illegible]	$-0.044^{+0.033}_{-0.034}$	$-0.011^{+0.013}_{-0.012}$	$0.0007^{+0.0037}_{-0.0037}$
[illegible]	[illegible]	< 0.257	< 0.241	< 0.120
$N_{\rm eff}$	$[illegible]_{-0.53}$	$2.92^{+0.36}_{-0.37}$	$2.89^{+0.36}_{-0.38}$	$2.99^{+0.34}_{-0.33}$
$Y_{\rm P}$	$0.246^{+0.039}_{-0.041}$	$0.240^{+0.024}_{-0.025}$	$0.239^{+0.024}_{-0.025}$	$0.242^{+0.023}_{-0.024}$
$dn_s/d\ln k$	$-0.004^{+0.015}_{-0.015}$	$-0.006^{+0.013}_{-0.013}$	$-0.005^{+0.013}_{-0.013}$	$-0.004^{+0.013}_{-0.013}$
$r_{0.002}$	< 0.102	< 0.107	< 0.101	< 0.106
w_0	$-1.56^{+0.60}_{-0.48}$	$-1.58^{+0.52}_{-0.41}$	$-1.57^{+0.50}_{-0.40}$	$-1.04^{+0.10}_{-0.10}$

The origins of dark energy, (dark matter, and baryon asymmetry) are major mystery of cosmology.

Dynamical or time-independent ?

Is dark energy dynamical (like inflation) or time-independent (Lambda, meta-stable state suggested by string landscape) ?



Parameter	*Planck*+SNe+BAO	*Planck*+BAO/RSD+WL
w_0	-0.961 ± 0.077	-0.76 ± 0.20
w_a	$-0.28^{+0.31}_{-0.27}$	$-0.72^{+0.62}_{-0.54}$
H_0 [km s^{-1}Mpc^{-1}]	68.34 ± 0.83	66.3 ± 1.8
σ_8	0.821 ± 0.011	$0.800^{+0.015}_{-0.017}$
S_8	0.829 ± 0.011	0.832 ± 0.013
$\Delta\chi^2$	-1.4	-1.4

$$w(a) = w_0 + w_a(1 - a)$$

There is no much strong constraint at present, though Lambda is consistent with observations.

If w_0 approaches minus unity within 1% by future observations, you may wonder if dark energy is almost Lambda-like and in a (meta)stable state.

But, this is not the case.

Dark energy view of inflation

(Ilic et al. arXiv:1002.4196)

Equation of state during inflation :

$$1 + w_\phi = \frac{2}{3}\epsilon_H, \quad \epsilon_H \equiv 2M_G^2\left(\frac{H'}{H}\right)^2 \simeq \epsilon \equiv \frac{1}{2}M_G^2\left(\frac{V'}{V}\right)^2$$

Tensor to scalar ratio : $r = 16\epsilon_H \lesssim 0.064$ (PLANCK with BICEP2/Keck Array BK14, 95%CL)

$\Rightarrow$ $1 + w_\phi \lesssim 0.0027.$

We have already had an example with w equal to -1 within 0.3% level.
However, it is not in a (meta)stable state but dynamical
because inflation must have ended to produce hot universe.

N.B. **Low scale inflation like new inflation** $\Rightarrow$ **$1+w_\varphi$: much smaller**

Dark energy view of inflation II

(Ilic et al. arXiv:1002.4196)

Scalar spectral index :

$$n_s - 1 = -4\epsilon_H + 2\eta_H, \quad \eta_H \equiv 2M_G^2 \frac{H''}{H} \simeq \eta - \epsilon, \quad \eta \equiv M_G^2 \frac{V''}{V}$$

$$n_s = 0.9649 \pm 0.0042$$ (PLANCK2018)

➡ **The spectral index excludes the HZ spectrum by more than 8 σ.**

➡ **The inflationary expansion significantly deviated from De Sitter expansion.**

Even if w is equal to -1 extremely, it does "not" necessarily mean that the present universe is in a (meta)stable state.

It can be dynamical and its expansion can deviate from De Sitter.

N.B. η can be large while ε, that is, $1+w_\varphi$ is small.

Next task is to identify the origin of the dark energy.

What kind of dynamical model of dark energy can we consider ?

Identification methods

- **Top down approach :**

  To construct the unique model from the ultimate theory like string theory.
  (Recently, it may not be so actively studied.)

- **Bottom up approach**

  To consider the most general model.
  Then, we can constrain models (or to single out the true model finally) from the observational results.

In this talk, we concentrate on the latter approach

Bottom up approach

- **Effective field theory approach :** **(Weinberg 2008, Cheung et al. 2008)**

  The low-energy effective theory (after integrating out heavy mode with its mass M).
  A ghost seems to appear around the cut-off scale M (>> E).

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{2}\frac{1}{M^2}(\Box\phi)^2$$

$\partial_\mu\phi\partial^\mu\phi \sim E^2\phi^2$, $\quad \frac{1}{M^2}(\Box\phi)^2 \sim \frac{E^4}{M^2}\phi^2$

(E: the energy scale we pay attention to)

$$M \gg E \quad \Longleftrightarrow \quad \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \gg \frac{1}{2}\frac{1}{M^2}(\Box\phi)^2$$

- **Most general theory without ghost**
  (if we are interested in the case in which higher derivative terms play an important role in the dynamics.)

In this talk, we concentrate on the latter approach

Integrating out a heavy field

σ : a heavy field with mass M, φ: a light field

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}M^2\sigma^2 - \partial_\mu\sigma\partial^\mu\phi$$

$$\wr$$

$$E^2\sigma^2 \ll M^2\sigma^2$$

$\uparrow$

energy scale we are interested in (E << M)

$$\sim -\frac{1}{2}M^2\sigma^2 + \sigma\Box\phi = -\frac{1}{2}M^2\left(\sigma - \frac{\Box\phi}{M^2}\right)^2 + \frac{1}{2}\frac{1}{M^2}(\Box\phi)^2$$

Integrating out σ

$$\sim \frac{1}{2}\frac{1}{M^2}(\Box\phi)^2$$

The following question arises:

What is the most general scalar-tensor theory without ghost ?

How widely can we extend scalar tensor theory ?

- **A kinetic term of an inflaton is not necessarily canonical.**

$$\mathcal{L} = X - V(\phi), \quad X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \quad \Longrightarrow \quad \mathcal{L} = K(\phi, X)$$

(k-inflation)

(Armendariz-Picon et.al. 1999)

- **An inflaton is not necessarily minimally coupled to gravity.**

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} M_G^2 R + \mathcal{L}_\phi \right) \quad \Longrightarrow \quad \Delta S = \int d^4x \sqrt{-g} f(\phi) R$$

(Brans-Dicke, Higgs inflation)

(Cervantes-Cota & Dehnen 1995, Bezrukov & M. Shaposhnikov 2008)

- **Action may include higher derivatives.** (Nicolis et.al. 2009)

$$\mathcal{L} = K(\phi, X) \quad \Longrightarrow \quad \Delta\mathcal{L} = G(\phi, X) \Box \phi$$

Theories with higher order derivatives are quite dangerous in general.

Lagrangian

Why does Lagrangian generally depend on only
a position q and its velocity dot{q} ?

Newton recognized that an acceleration, which is given by the second time derivative of a position, is related to the Force :

$$m\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} = \boldsymbol{F}\left(\boldsymbol{x}, \dot{\boldsymbol{x}}\right).$$

The Euler-Lagrange equation gives an equation of motion up to the second time derivative if a Lagrangian is given by L = L(q,dot{q},t).

$$\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0, \quad \Longrightarrow \quad \ddot{q} = \ddot{q}\left(\dot{q}, q\right) \quad \Longrightarrow \quad q(t) = Q\left(\dot{q}_0, q_0, t\right).$$

(if $p := \frac{\partial L}{\partial \dot{q}}$ depends on dot{q} ⇔ non-degenerate condition.)

What happens if Lagrangian depends on higher derivative terms ?

Example with higher order (time) derivatives

- $L = \frac{1}{2}\ddot{q}^2(t)$ $\Longrightarrow$ (EL eq.) $q^{(4)} = 0$ **requires 4 initial conditions.**

  $\Updownarrow$

  2 (real) DOF

- $L_{\rm eq}^{(1)} = \ddot{q}u - \frac{1}{2}u^2$ $\Longrightarrow$ (EL eq.) $\begin{cases} \ddot{u} = 0, \\ \ddot{q} = u, \end{cases}$ $\Longrightarrow$ $q^{(4)} = 0$

$$x \equiv \frac{q-u}{\sqrt{2}}, \quad y \equiv \frac{q+u}{\sqrt{2}} \Longrightarrow L_{\rm eq}^{(1)} = -\dot{q}\dot{u} - \frac{1}{2}u^2 = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\dot{y}^2 - \frac{1}{4}(x-y)^2.$$

$$\Longrightarrow H = \frac{1}{2}p_x^2 - \frac{1}{2}p_y^2 + \frac{1}{4}(x-y)^2.$$

$$(p_x \equiv \dot{x}, \quad p_y \equiv \dot{y})$$

2 (real) DOF = 1 healthy & 1 ghost

- $L_{\rm eq}^{(2)} = \frac{1}{2}\dot{Q}^2 + \lambda(Q - \dot{q})$ $\Longrightarrow$ $p \equiv \frac{\partial L_{\rm eq}^{(2)}}{\partial \dot{q}} = -\lambda, \quad P \equiv \frac{\partial L_{\rm eq}^{(2)}}{\partial \dot{Q}} = \dot{Q}.$

$$\Longrightarrow H = p\dot{q} + P\dot{Q} - L_{\rm eq}^{(2)} = \frac{1}{2}P^2 + pQ.$$

Hamiltonian is unbounded through a linear momentum !!

Ostrogradski's theorem

(Ostrogradsky 1850)

Assume that $L = L(\ddot{q}, \dot{q}, q)$ **and** $\frac{\partial L}{\partial \ddot{q}}$ **depends on** $\ddot{q}$ **:**

(Non-degeneracy)

$$\Rightarrow \quad \frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) + \frac{\mathrm{d}^2}{\mathrm{d}^2 t}\left(\frac{\partial L}{\partial \ddot{q}}\right) = 0, \quad \Longrightarrow \quad q^{(4)} = q^{(4)}\left(q^{(3)}, \ddot{q}, \dot{q}, q\right).$$

Canonical variables :

$$\begin{cases} q, & p := \frac{\partial L}{\partial \dot{q}} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \ddot{q}} \left(= \frac{\partial L_{\mathrm{eq}}}{\partial \dot{q}}\right), \\ Q := \dot{q}, & P := \frac{\partial L}{\partial \ddot{q}} \left(= \frac{\partial L_{\mathrm{eq}}}{\partial \dot{Q}}\right). \end{cases}$$

$$L_{\mathrm{eq}} = L(\dot{Q}, \dot{q}, q) + \lambda(Q - \dot{q})$$

Non-degeneracy $\Leftrightarrow \quad \ddot{q} = \ddot{q}\left(q, \dot{q}, \frac{\partial L}{\partial \ddot{q}}\right) \quad \Leftrightarrow \quad \dot{Q} = \ddot{q} = \ddot{q}\,(q, Q, P)$

Hamiltonian:

$$\begin{aligned} H(q, Q, p, P) &:= p\dot{q} + P\dot{Q} - L \\ &= pQ + P\ddot{q}(q, Q, P) - L(q, Q, \ddot{q}(q, Q, P)). \end{aligned}$$

$\Rightarrow$ **p depends linearly on H so that no system of this form can be stable !!**

N.B. $\frac{\partial L}{\partial \phi} - \partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu \phi)}\right) + \partial_\mu \partial_\nu\left(\frac{\partial L}{\partial(\partial_\mu \partial_\nu \phi)}\right) = 0. \Rightarrow \frac{i}{(p^2 + m_1^2)(p^2 + m_2^2)} = \frac{1}{m_2^2 - m_1^2}\left(\frac{i}{p^2 + m_1^2} - \frac{i}{p^2 + m_2^2}\right).$

(propagators)

How to circumvent Ostrogradsky's arguments to obtain healthy higher order derivative theories ?

Loophole of Ostrogradski's theorem

We can break the non-degeneracy condition which requires that $\frac{\partial L}{\partial \ddot{q}}$ depends on ddot{q}.

(NB: another interesting possibility is infinite derivative theory)

In case Lagrangian depends on only a position q and its velocity dot{q}, degeneracy implies that EOM is first order, which represents not the dynamics but the constraint.

$$\Updownarrow$$

In case Lagrangian depends on q, dot{q}, ddot{q}, degeneracy implies that EOM can be (more than) second order, which can represent the dynamics.

Generalized Galileon = Horndeski

Deffayet et al. 2009, 2011 Horndeski 1974

equivalence

Kobayashi, MY, Yokoyama 2011

$$\begin{cases} \mathcal{L}_2 = K(\phi, X), \\ \mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \\ \mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ \qquad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{cases}$$

$$X = -\frac{1}{2}(\nabla\phi)^2, \qquad G_{iX} \equiv \partial G_i/\partial X.$$

This is the most general scalar tensor theory whose Euler-Lagrange EOMs are up to second order though the action includes second derivatives.
Many of inflation and dark energy models can be understood in a unified manner.

NB :
- **$G_4 = M_G^2 / 2$ yields the Einstein-Hilbert action**
- **$G_4 = f(\varphi)$ yields a non-minimal coupling of the form $f(\varphi)R$**
- **The new Higgs inflation with $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ comes from $G_5 \propto \varphi$ after integration by parts.**

Horndeski theory

Horndeski 1974

In 1974, Horndeski presented the most general action (in four dimensions) constructed from the metric g, the scalar field φ, and their derivatives, $\partial g_{\mu\nu}, \partial^2 g_{\mu\nu}, \partial^3 g_{\mu\nu}, \cdots, \partial\phi, \partial^2\phi, \partial^3\phi, \cdots$ still having second-order equations.

$$\mathcal{L}_H = \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\left[\kappa_1\nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} + \frac{2}{3}\kappa_{1X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi + \kappa_3\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma} + 2\kappa_{3X}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi\right]$$
$$+\delta^{\alpha\beta}_{\mu\nu}\left[(F+2W)R_{\alpha\beta}{}^{\mu\nu} + 2F_X\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_8\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\right] - 6\left(F_\phi + 2W_\phi - X\kappa_8\right)\Box\phi + \kappa_9.$$

κ1, κ3, κ8, κ9, F : functions of φ & X with

W = W(φ)

$$\delta^{\alpha_1\alpha_2\ldots\alpha_n}_{\mu_1\mu_2\ldots\mu_n} = n!\delta^{[\alpha_1}_{\mu_1}\delta^{\alpha_2}_{\mu_2}\ldots\delta^{\alpha_n]}_{\mu_n}.$$

$$F_X = 2(\kappa_3 + 2X\kappa_{3X} - \kappa_{1\phi}).$$

What is the relation between Generalized Galileon and Horndeski's models ?

⇒ **Both models are completely equivalent :** **Kobayashi, MY, Yokoyama 2011**

$$\begin{cases} K = \kappa_9 + 4X\int^X dX'\left(\kappa_{8\phi} - 2\kappa_{3\phi\phi}\right), \\ G_3 = 6F_\phi - 2X\kappa_8 - 8X\kappa_{3\phi} + 2\int^X dX'(\kappa_8 - 2\kappa_{3\phi}), \\ G_4 = 2F - 4X\kappa_3, \\ G_5 = -4\kappa_1, \end{cases}$$

$$\begin{cases} \mathcal{L}_2 = K(\phi, X), \\ \mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \\ \mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ \quad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{cases}$$

Cosmological perturbations in Horndeski theory

Kobayashi, MY, Yokoyama 2011

- **Tensor perturbations:**

$$S_T^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\nabla h_{ij})^2\right].$$

$$\begin{cases} \mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right], \\ \mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right] \end{cases} \qquad c_T^2 := \frac{\mathcal{F}_T}{\mathcal{G}_T}$$

If this Horndeski field is responsible for dark energy, the sound velocity of tensor perturbations (GWs) must be very close to unity.

(e.g. Creminelli & Vernizzi 2017)
(Kimura & Yamamoto 2012)

$$c_T^2 = c_{\mathrm{GW}}^2 \simeq 1.$$

(GW170817 & GRB170817A)

(gravitational Cherenkov radiation)

$$\begin{cases} \mathcal{L}_2 = K(\phi, X), \\ \mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \\ \mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ \qquad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{cases}$$

→

$$G_{4X} \simeq 0, \quad G_5 \simeq 0$$

Summary

- Though the dark energy is consistent with cosmological constant, it is still too early to conclude it. The dynamical model is still worth studying.
- It is quite useful to consider a general model because it can accommodate many models in a unified way.
- One of the most famous examples is Horndeski theory, which is the most general (single) scalar-tensor theory whose EL equations are up to the second order.
- The future observations including GWs will strongly constrain models.