Outline

I. **Conformal and disformal transformations**

II. **RSD in Standard Cosmology and Kaiser Formula**

III. **Coupled Dark Matter and Modified Kaiser Formula**

IV. **Concrete Cosmological Examples**

V. **Forecasts From Future Galaxy Surveys**

Disformal transformations Bekenstein (1993)

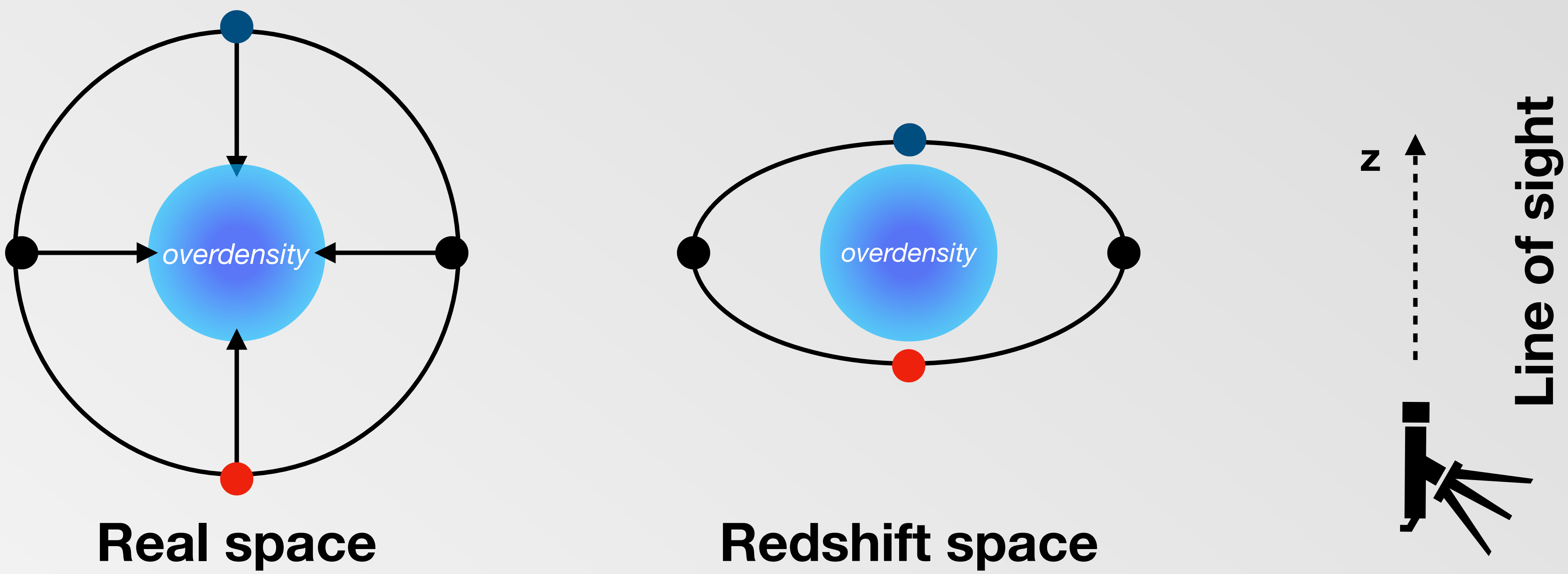
$$\tilde{g}_{\mu\nu} = A(\phi, X) g_{\mu\nu} + B(\phi, X) \partial_\mu \phi \partial_\nu \phi$$

$$X \equiv -\frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi$$

- Do not introduce second-order terms in the dynamical equations Bettoni+ (2013)
- Generates Beyond-Horndeski terms Bettoni+ (2013)
- Doesn't change the number of physical d.o.f. Demenech+ (2015)
- Can distort the causal structure between the two space-times

- Special disformal transformations $\tilde{g}_{\mu\nu} = A(\phi) g_{\mu\nu} + B(\phi) \partial_\mu \phi \partial_\nu \phi$
  - Eliminates non-minimal coupling in DBI Galileon theory Zumalacarregui+(2013)
  - Horndeski's theory: invariant under such transformation Bettoni+ (2013)
  - Recently, has been used in a coupled DM model Gleyzes+ (2016)

Redshift space distortions



From real to redshift space:

galaxy's peculiar velocity along the line of sight

Position

$$\mathbf{s} = \mathbf{x} + \frac{v_{\mathrm{g,z}}}{aH}\hat{z}$$

$n_{\mathrm{g}} = \bar{n}_{\mathrm{g}}(1+\delta_g)$

n_{g} : *number density*

$\bar{n}_{\mathrm{g}}$: *ave. number density*

Density contrast

$$\delta_{\mathrm{g,s}} = \delta_{\mathrm{g}} - \frac{\nabla_z v_{\mathrm{g,z}}}{aH}$$

$\delta_{\mathrm{g}} = b_g \delta_{\mathrm{m}}$ (linear bias)

$v_{\mathrm{g}} \approx v_{\mathrm{m}}$

Standard cosmology: linear perturbation

Newtonian gauge:

$$ds^2 = -[1+2\Phi(t,\mathbf{x})]dt^2 + a^2(t)[1-2\Psi(t,\mathbf{x})]d\mathbf{x}^2$$

Sub-horizon approximation: $\frac{k}{aH} \gg 1$

Poisson, continuity and Euler equations (in Fourier space):

$$\frac{k^2}{a^2}\Phi = -4\pi G\rho_{\rm m}\delta_{\rm m}$$

$$\dot{\delta}_{\rm m} + \frac{k^2}{a^2}v_{\rm m} = 0$$

$$\dot{v}_{\rm m} - \Phi = 0$$

EoM for the density contrast

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\delta_m = 0$$

Velocity equation

$$v_{\rm m} = -\frac{a^2 H}{k^2} f_{\rm m}\delta_{\rm m}$$

Baryons and DM follow similar equations

Growth function $\delta_{\rm m}(t,k) = D_{\rm m}(t)\delta_0(k)$

δ_0 : Initial density contrast

Linear growth rate $f_{\rm m}(t) = \frac{d\log D_{\rm m}}{d\log a}$

Standard cosmology: power spectrum

Dependence on the velocity

Kaiser formula

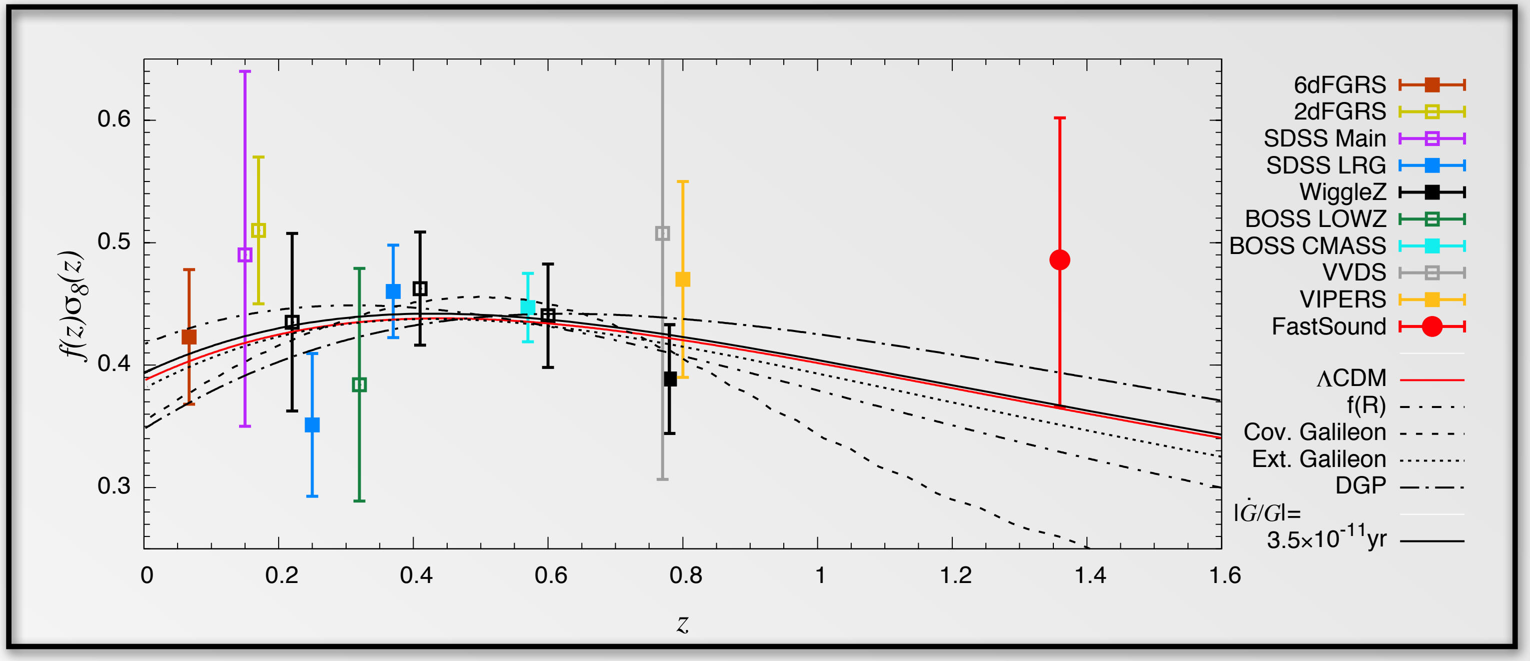
$\mu = \cos\theta = \hat{k}\cdot\hat{z}$

$$P_{\mathrm{g,s}}(\mathbf{k};t) = \left[b_{\mathrm{g}} + \mu^2 f_m(t)\right]^2 P_{\mathrm{m}}(k;t)$$ (Kaiser, 1987)

Galaxy PS in redshift space

Matter PS in real space

Linear growth rate measurable by RSD



Okumura et al. (2016)

Non-minimally coupled dark matter: set-up

Action:

$$S = \int dx^4 \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2} R + K(\phi, X)\right] + S_M$$

GR DE Matter

$$S_{\rm m} = S_{\rm b} + S_{\rm c}$$

Baryons: constraints from solar system experiments => **minimal coupling**

$$S_{\rm b} = \int {\rm d}^4x \sqrt{-g}\, \mathcal{L}_{\rm b}[g, \psi_{\rm b}]$$

CDM: unknown fundamental nature => **non-minimal coupling**

$$S_{\rm c} = \int {\rm d}^4x \sqrt{-\tilde{g}}\, \mathcal{L}_{\rm c}[\tilde{g}, \psi_c]$$

$$\tilde{g}_{\mu\nu} = A(\phi, X) g_{\mu\nu} + B(\phi, X)\partial_\mu\phi\partial_\nu\phi$$

conformal factor — disformal factor

Metric coupling

Basic equations

Einstein

$$G_{\mu\nu} = \frac{1}{\mathrm{M}_{\mathrm{Pl}}^2}\left(T_{\mu\nu}^{(\mathrm{b})} + T_{\mu\nu}^{(\mathrm{c})} + T_{\mu\nu}^{(\phi)}\right)$$

Energy-momentum conservation

$$\nabla^\mu T_{\mu\nu}^{(\mathrm{b})} = 0 \qquad \nabla^\mu\left(T_{\mu\nu}^{(\mathrm{c})} + T_{\mu\nu}^{(\phi)}\right) = 0$$

E.o.M.

$$\nabla^\mu T_{\mu\nu}^{(\mathrm{c})} = -Q\phi_\nu$$

$$\Box\phi - V_\phi = Q$$

$$Q = -\frac{1}{\sqrt{-g}}\frac{\delta\left(\sqrt{-\tilde{g}}\mathscr{L}_{\mathrm{c}}\right)}{\delta\phi} = \nabla_\mu W^\mu - Z$$

$$Z = \frac{1}{2A}\left[\left\{A_\phi + \frac{A_X X\left(A_\phi - 2B_\phi X\right)}{A - A_X X + 2B_X X^2}\right\}T_{(c)} + \left\{B_\phi + \frac{B_X X\left(A_\phi - 2B_\phi X\right)}{A - A_X X + 2B_X X^2}\right\}T_{(c)}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$$

$$W^\mu = \frac{1}{2A}\left[2BT_{(\mathrm{c})}^{\mu\nu}\partial_\nu\phi - \frac{A - 2BX}{A - A_X X + 2B_X X^2}\times\left(A_X T_{(\mathrm{c})} + B_X T_{(\mathrm{c})}^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right)\partial^\mu\phi\right]$$

Background evolution

Evolution equations

$$\dot{\rho}_{\rm c} + 3H\rho_{\rm c} = Q_0\dot{\phi}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -Q_0$$

Background coupling

$$Q_0 = \frac{\rho_{\rm c}}{\dot{\phi}}\frac{{\rm d}\Gamma}{{\rm dt}}, \qquad \Gamma(\phi, X) = \ln\left[\frac{2A - A_X\dot{\phi}^2 + B_X\dot{\phi}^4}{\sqrt{A - B\dot{\phi}^2}}\right]$$

$$\rho_{\rm c} \propto a^{-3}e^{\Gamma(\phi,X)} \quad \Rightarrow \quad m(\phi, X) = m_0 e^{\Gamma - \Gamma_0}$$

At classical level, interaction can be interpreted as a variation in the coupled particle's mass

Coupled DM: linear perturbation

(sub-horizon + quasistatic approx.)

- **Poisson** and **baryon** equations: unchanged

- Continuity and Euler equations for **CDM**: modified

Effective gravitational constant

EoM for the CDM density contrast

$$\ddot{\delta}_c + 2H_{\rm eff}\dot{\delta}_c - 4\pi G_{\rm eff}\rho_m\delta_m = 0$$

Effective Hubble friction

- CDM velocity field

$$v_c(t,\mathbf{k}) = -\frac{a^2 H}{k^2} f_c^{\rm eff}(t)\,\delta_c(t,\mathbf{k})$$

Effective growth rate

Under the hood: perturbed equations

Continuity & Euler:

$$\dot{\delta}_c + \frac{k^2}{a^2} v_c = \frac{\dot{\phi}}{\rho_c}\Big(\delta Q - Q_0 \delta_c\Big)\,,$$
$$\dot{v}_c - \Phi = \frac{Q_0}{\rho_c}\Big(\delta\phi - \dot{\phi} v_c\Big)\,.$$

EoM for the scalar field (QS approx.)

$$-\mathcal{A}\frac{k^2}{a^2}\delta\phi \equiv -\left(\mathcal{A}_1 \frac{k^2}{a^2} + m_\phi^2\right)\delta\phi = \delta Q\,,$$

$$\delta Q = (R_1 + R_2)\dot{\phi}\dot{\delta}_c + Q_0\delta_c + R_1\dot{\phi}\frac{k^2}{a^2} v_c + R_2 \frac{k^2}{a^2}\delta\phi\,.$$

EoM for the CDM density contrast

$$(1 - \Upsilon_1 - \Upsilon_2)\,\ddot{\delta}_c + 2H(1 - \mathcal{E}_1)\,\dot{\delta}_c - 4\pi G\Big[(1 - \mathcal{E}_2)\,\rho_c\delta_c + (1 - \Upsilon_1)\,\rho_b\delta_b\Big] = 0$$

$$\mathcal{E}_1 = \Upsilon_1 + \Upsilon_2 - \frac{Q_0\dot{\phi}}{2\rho_c H}(1 - \Upsilon_1 - \Upsilon_2) + \frac{1}{2H}\left(\dot{\Upsilon}_2 + \frac{\dot{\Upsilon}_1\Upsilon_2}{1-\Upsilon_1}\right)\,,$$

$$\mathcal{E}_2 = \Upsilon_1 + M_{\rm Pl}^2\left[\frac{2\dot{Q}_0\dot{\phi}}{\rho_c^2}\frac{\Upsilon_2}{\Upsilon_3} + \frac{Q_0\dot{\phi}}{\rho_c^2\Upsilon_3}\left(2\dot{\Upsilon}_2 + \Upsilon_2\left(\frac{2\dot{\Upsilon}_1}{1-\Upsilon_1} - \frac{2(\ddot{\phi} - 2H\dot{\phi})}{\dot{\phi}}\right)\right) - \frac{2Q_0\dot{\phi}^3}{\rho_c^3}\frac{\dot{\mathcal{A}}\Upsilon_2}{\Upsilon_3^2} + \frac{2Q_0^2\dot{\phi}^2}{\rho_c^3\Upsilon_3^2}\left(\Upsilon_2 + \Upsilon_1\Upsilon_3 + \Upsilon_2\Upsilon_3 - \Upsilon_3\right)\right].$$

$$\Upsilon_1 = \frac{\dot{\phi}^2}{\rho_c}\frac{\mathcal{A}R_1}{\mathcal{A}+R_2}\,,\quad \Upsilon_2 = \frac{\dot{\phi}^2}{\rho_c}\frac{\mathcal{A}R_2}{\mathcal{A}+R_2}\,,\quad \Upsilon_3 = \frac{\mathcal{A}\dot{\phi}^2}{\rho_c}\,.$$

Coupled DM: linear perturbation

CDM's continuity & Euler eqs are modified

Evolution of total matter is modified

$$\rho_m \delta_m = \rho_b \delta_b + \rho_c \delta_c$$

$$\rho_m v_m = \rho_b v_b + \rho_c v_c$$

RSD probes velocity, so….

Velocity field

$$v_{\rm m}(t, \mathbf{k}) = -\frac{a^2 H}{k^2} f_{\rm m}^{\rm eff}(t)\, \delta_{\rm m}(t, \mathbf{k})$$

$$f_{\rm m}^{\rm eff} = f_{\rm m} + \Delta f_{\rm m}^{\rm eff}$$

Actual growth rate

$$f_{\rm m} = \frac{{\rm d} \log D_{\rm m}}{{\rm d} \log a}$$

DM-DE coupling effect depending on the conformal and disformal functions

Coupled DM: effective growth rate

Cold dark matter

$$f_c^{\rm eff} = f_c - \underbrace{\frac{\Upsilon_2}{1-\Upsilon_1}\left(f_c - \frac{Q_0}{H\dot{\phi}}\right)}_{\Delta f_c}$$

$$\Upsilon_2 = \Upsilon_2(A_X, B_X, \dots)$$

$$A(\phi), B(\phi) \Rightarrow \Delta f_c = 0$$

Total matter

$$f_m^{\rm eff} = \frac{\omega_c D_c f_c^{\rm eff} + \omega_b D_b f_b^{\rm eff}}{\omega_c D_c + \omega_b D_b} = f_m + \underbrace{\omega_c \frac{D_c}{D_m}\Delta f_c - \omega_b \frac{Q_0 \dot{\phi}}{H\rho_m}\frac{D_c - D_b}{D_m}}_{\Delta f_m}$$

X-dependent contribution

BG coupling

Coupled DM: power spectrum

Modified Kaiser formula

$$v_m \propto f_m^{\rm eff}$$
$$f_m^{\rm eff} = f_m + \Delta f_m$$

$$P_{\rm g,s}(\mathbf{k};t) = \left[b_{\rm g} + \mu^2 f_{\rm m}^{\rm eff}(t)\right]^2 P_{\rm m}(k;t)$$

Standard scenario (minimally coupled DM)		
Growth function & growth rate	$D_{\rm m} = D_{\rm c} = D_{\rm b}$	$f_{\rm m}^{\rm eff} = f_{\rm m}$
Coupled DM	**RSD measures the effective growth rate (actual growth rate + DM-DE interaction)**	

Discriminating growth and coupling with RSD:
- multiple z measurements /
- Cross correlation with other proves (e.g. weak lensing)

Concrete examples

- Canonical scalar field

$$\mathcal{L}_\phi = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)$$

$$V(\phi) = V_0\phi^{-n}, \quad n > 0$$

$M_{\rm Pl} = 1$

- Coupling

$$\tilde{g}_{\mu\nu} = A g_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi$$

α : *coupling constant*

Model I (conf.)	$A(\phi) = e^{-2\alpha\phi}$,	$B = 0$	$\longrightarrow$	(Amendola, 2000)
Model II (conf.)	$A(\phi) = e^{-\alpha\phi^2}$,	$B = 0$	$\longrightarrow$	Tracking behavior
Model III (disf.)	$A = 1$,	$B(X) = \dfrac{\alpha}{X}$	$\longrightarrow$	No interaction at background level

Model I: perturbation

Evolution of perturbation $$\ddot{\delta}_c + 2H\left(1-\mathscr{E}_1\right)\dot{\delta}_c - 4\pi G\left[\left(1-\mathscr{E}_2\right)\rho_c\delta_c + \rho_b\delta_b\right] = 0$$

Effective H $$\mathscr{E}_1 = \frac{\alpha}{2}\frac{\dot{\phi}}{H}$$ $\alpha > 0$ => *less friction*

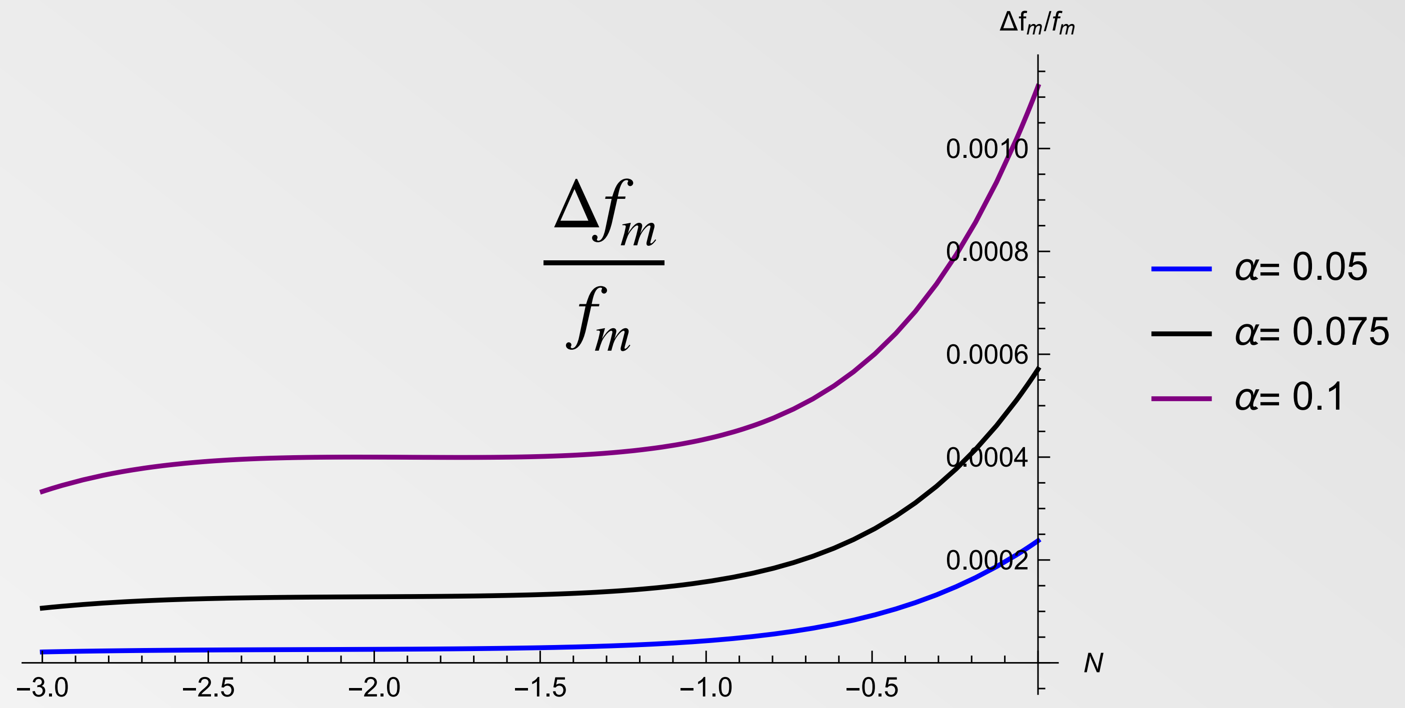
Effective G $$\mathscr{E}_2 = -2\alpha^2$$ => *scalar fifth-force (attractive)*

=> Growth enhancement

$\mathscr{E}_2$ *constant:*

interaction affects growth even at early times

$$\frac{\Delta f_m}{f_m}$$



Model II: perturbation

Evolution of perturbation $$\ddot{\delta}_c + 2H\left(1-\mathcal{E}_1\right)\dot{\delta}_c - 4\pi G\left[\left(1-\mathcal{E}_2\right)\rho_c\delta_c + \rho_b\delta_b\right] = 0$$

Effective H $$\mathcal{E}_1 = \frac{\alpha}{2}\frac{\phi\dot{\phi}}{H}$$ $\alpha > 0$ => *less friction*

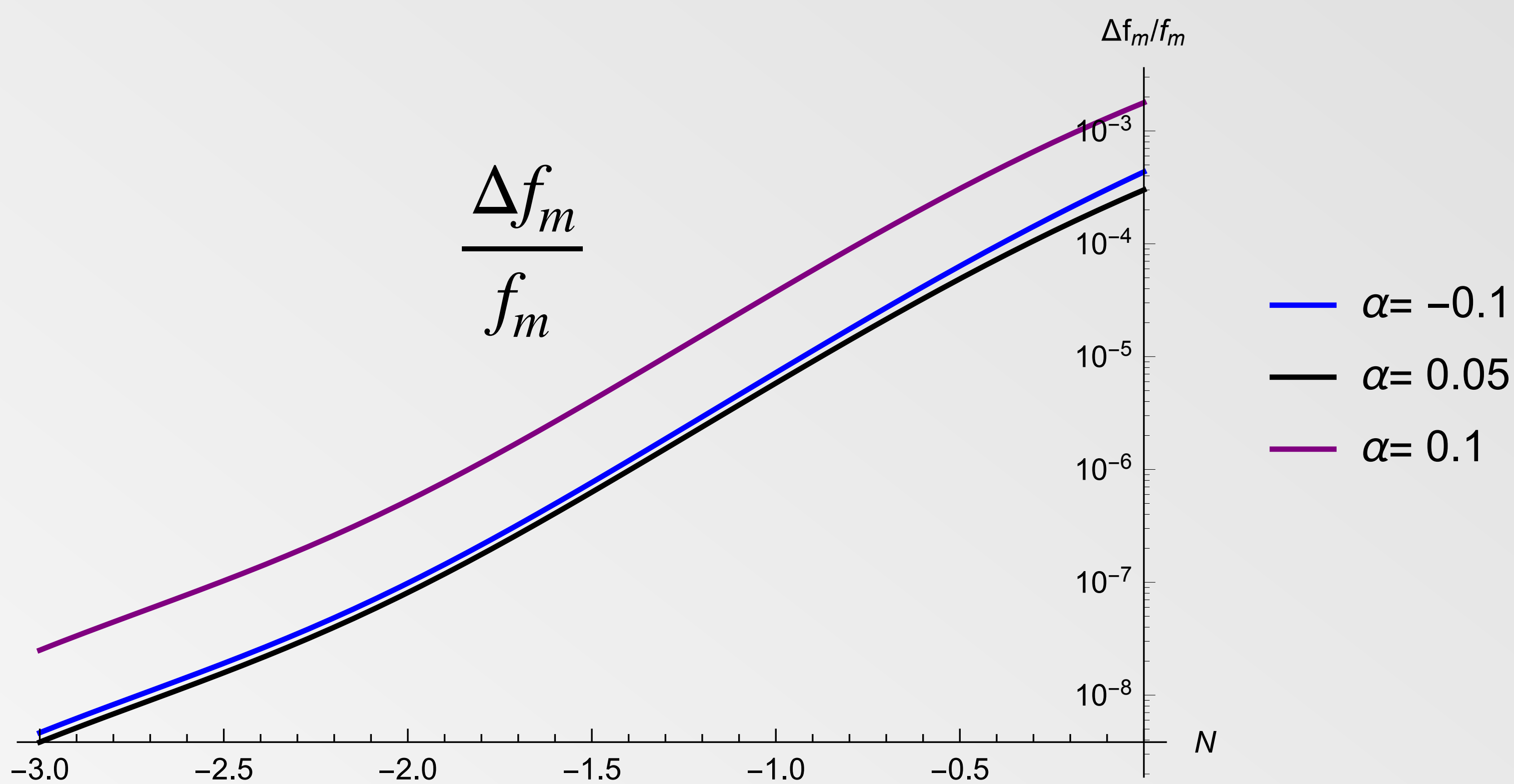
Effective G $$\mathcal{E}_2 = -2\alpha^2\phi^2$$ => *scalar fifth-force (attractive)*

=> Growth enhancement

At early times: $\mathcal{E}_1 \approx \mathcal{E}_2 \approx 0$

interaction affects only late-time behavior

$$\frac{\Delta f_m}{f_m}$$



Model III: disformal coupling

Conformal/ disformal functions

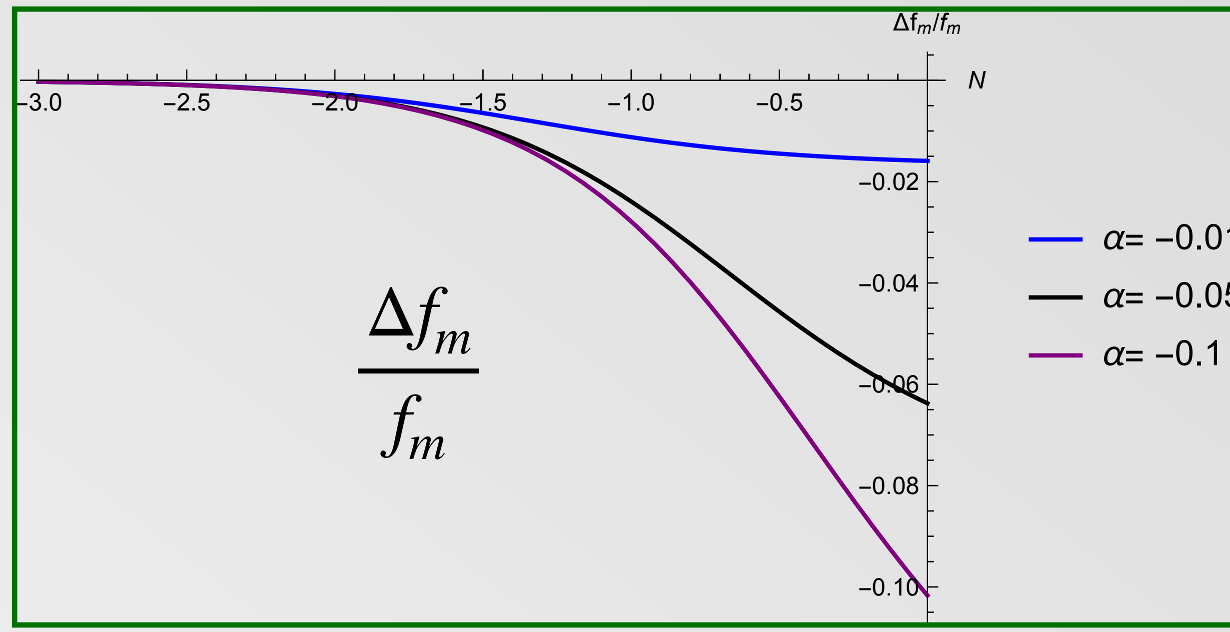
$$A(\phi) = 1, \quad B = \frac{\alpha}{X}$$

$$\ddot{\delta}_c + 2H\left(1 - \mathscr{E}_1\right)\dot{\delta}_c - 4\pi G\left(1 - \mathscr{E}_2\right)\rho_m\delta_m = 0$$

$Q_0 = 0$ **Coupling only at pert. level**

Effective H

$$\mathscr{E}_1 = -\frac{2\alpha^2\rho_c\dot{\phi}\left(3\dot{\phi} + 2V_\phi\right)}{H\left(\dot{\phi}^2 - 2\alpha\rho_c\right)\left[(1-2\alpha)\dot{\phi}^2 - 2\alpha\rho_c\right]}$$



Effect of disformal coupling: Suppression of growth

Effective G

$$\mathscr{E}_2 = \frac{2\alpha\dot{\phi}^2}{\dot{\phi}^2 - 2\alpha\rho_c}$$

$$c_s^2 > 0 \Rightarrow \alpha < \frac{\dot{\phi}^2}{2\rho_c}$$

Forecast: Fisher matrix analysis

Galaxy survey:

- Phase two of the Square Kilometer Array project (SKA2)
  - Network of radio telescopes in South Africa and Australia
  - Beginning: 2025
  - Survey specifications based on Yahya et al. (2015)
- Euclid space satellite
  - Estimated launch: 2021
  - Survey specifications based on Amendola et al. (2018)

Fisher matrix analysis:

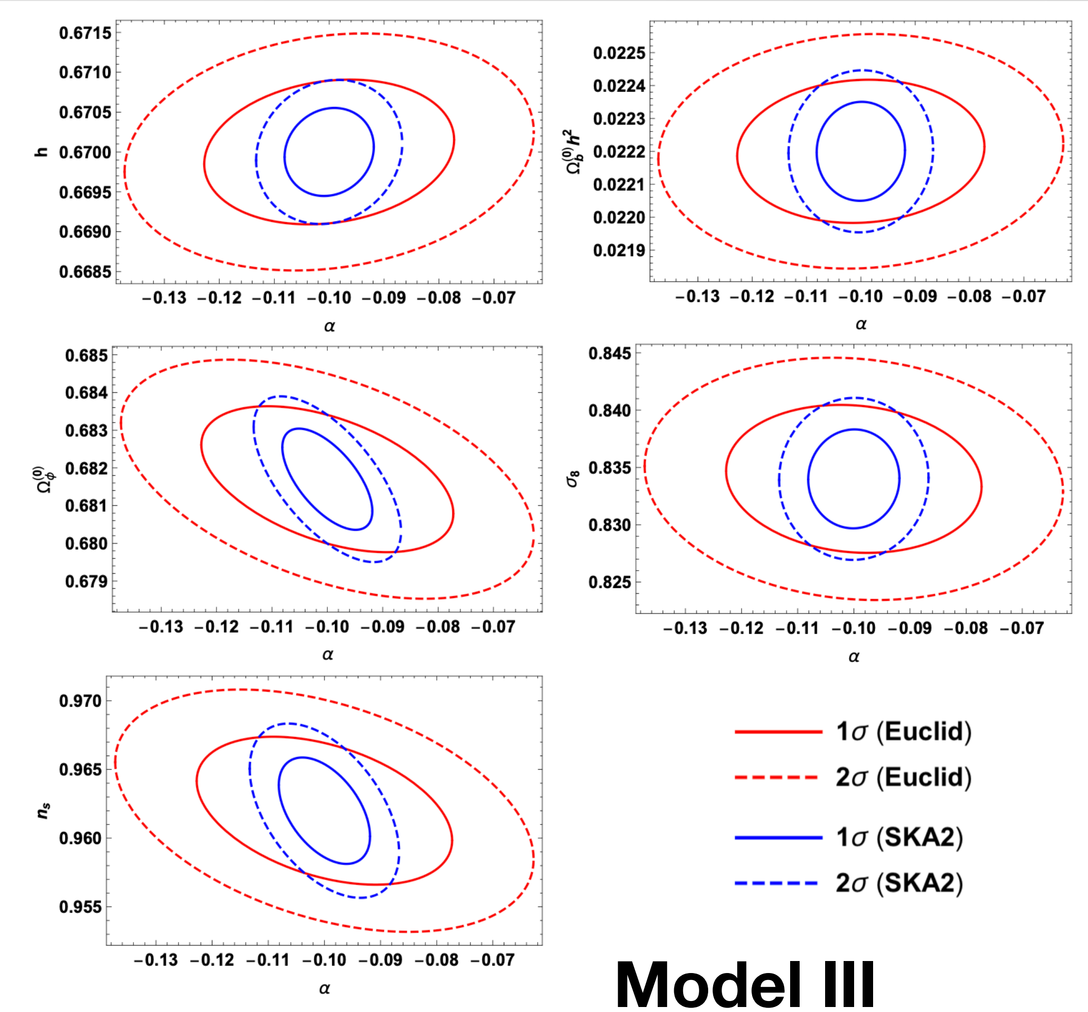
Estimating the constraints from a future experiment

$$F_{\alpha\beta} = \sum_{z_i} \int_{k_{\rm min}}^{k_{\rm min}} \frac{{\rm d}^3\mathbf{k}}{(2\pi)^3} V_{\rm eff}(\mathbf{k}, z_i) \frac{\partial \ln P_{\rm obs}(\mathbf{k}, z_i)}{\partial \theta^\alpha} \frac{\partial \ln P_{\rm obs}(\mathbf{k}, z_i)}{\partial \theta^\beta}$$

$$P_{\rm obs}(\vec{k}, z) = \mathcal{N}_{\rm AP}(z) \left(b + f_{\rm m}^{\rm eff}\mu^2\right)^2 P_{\rm m}(k, z) e^{-k^2\mu^2\sigma_{\rm NL}^2}.$$

Forecast: results

Model	α (Fid.)	Survey	$10^3 \times \sigma(\alpha)$	
I	0.04	Euclid	5.90	~15%
		SKA2	3.72	~ 9%
II	0.04	Euclid	26.37	~66%
		SKA2	19.99	~50%
III	-0.1	Euclid	15.00	~15%
		SKA2	5.36	~5%



Model III

Table 9. Marginalized mean values and 68% C.L. intervals for coupled DE (see Sect. 5.3.4).

CDE models	*Planck* TT+lowP	*Planck* TT+lowP +BSH	*Planck* TT+lowP +WL	*Planck* TT+lowP +BAO/RSD	*Planck* TT+lowP +WL+BAO/RSD
β	<0.066 (95%)	$0.037^{+0.018}_{-0.015}$	$0.043^{+0.026}_{-0.022}$	$0.034^{+0.019}_{-0.016}$	$0.037^{+0.020}_{-0.016}$
α	$0.43^{+0.15}_{-0.33}$	$0.29^{+0.077}_{-0.26}$	$0.44^{+0.18}_{-0.20}$	$0.40^{+0.15}_{-0.20}$	$0.45^{+0.17}_{-0.23}$
H_0 (km s^{-1} Mpc^{-1}) .	$65.4^{+3.2}_{-2.6}$	$67.47^{+0.88}_{-0.79}$	67.6 ± 2.8	66.7 ± 1.1	66.9 ± 1.1
σ_8	$0.812^{+0.031}_{-0.026}$	0.829 ± 0.018	$0.819^{+0.031}_{-0.026}$	0.817 ± 0.017	0.810 ± 0.017

~50%

Ade et al. (2016)

Summary

- Conformal/disformal metric couplings: possibility of defining all evolution equations without specifying the DM component

- Effective growth rate: measured by RSD

    - combination of the actual growth rate & DM-DE interaction effects

- Future galaxy surveys: possibility of putting constraints on the interaction (~9%)

Thank you!