

# *Unified Superfluid Dark Sector*

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*EF, Justin Khoury, Robert Brandenberger and G. Franzmann. 1810.09474, JCAP.*

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University of São Paulo, February 2019

# The model

Different approach to the problem!

New model that **unifies** into a **single** framework:

- ✓ Dark energy
- ✓ Cold dark matter
- ✓ Small scale behavior of dark matter



Minimal: **DM only** model

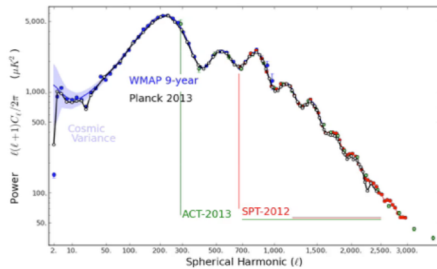
This can be done using the physics of **superfluidity**!

Predictions that can be tested.

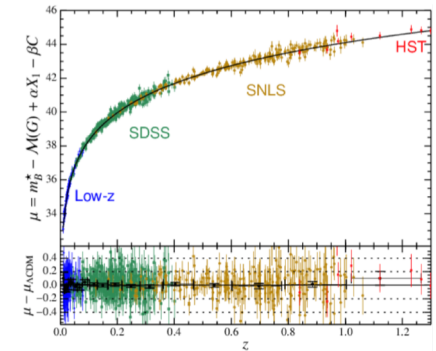
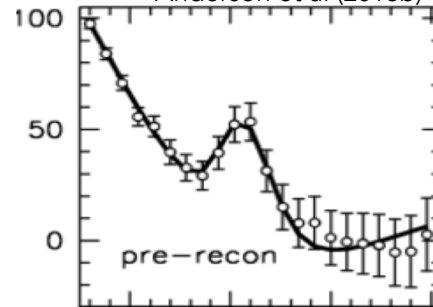


# Dark Matter and Dark Energy

Spergel



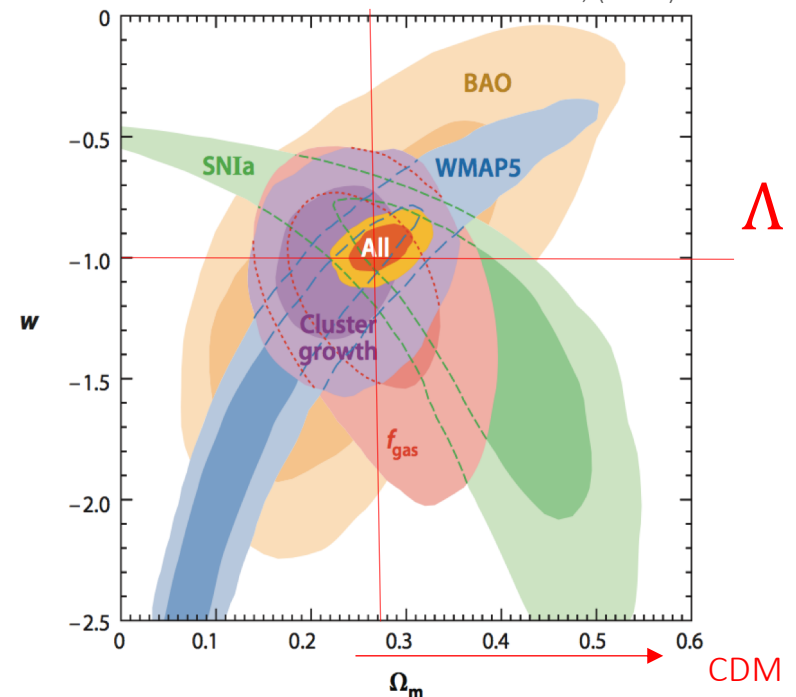
Anderson et al (2013b)



## $\Lambda$ CDM: Simple, yet exotic...

- Dark matter: collisionless, cold, pressureless particle
- Dark energy : cosmological constant  $\Lambda$  drives cosmic acceleration.

Allen et al, (2013)



# Challenges

## Dark Energy: $\Lambda$

- Fine tuning:

$$\rho_{\Lambda}^{th} = \begin{cases} 10^{76} GeV^4 & (Planck) \\ 10^{64} GeV^4 & (GUT) \end{cases}$$

vs.

$$\rho_{\Lambda}^{obs} \sim \rho_{c,0} \sim 10^{-47} GeV^4$$

THEORY

- Radiative instability: at each order in perturbation theory, repeatedly fine tune required.

## Dark Matter: Small scales

OBSERVATIONS

- Galactic scales
- Regularity/diversity of rotation curves

✓ BTFR

$$V_f^4 = \mathbf{a_0} G_N M_b$$

✓ Radial acceleration relation (RAR)

✓ ...

## Dark Energy:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

acceleration      gravity      cosmological constant

slows down expansion      speeds up expansion

- Dark Energy: new fluid with  $p < 0$ .
- Modified Gravity

Dark Matter: **Large Scales:** standard cold DM particle

## Small scales

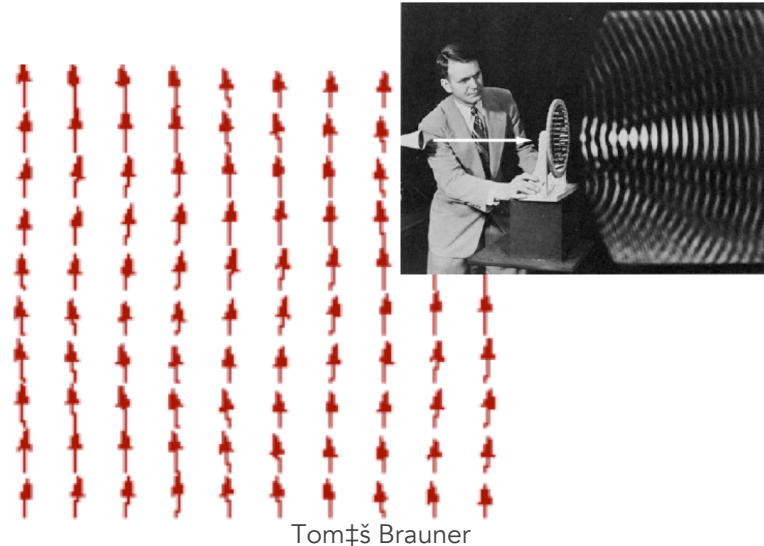
$$a = \begin{cases} a_N & a_N \gg a_0 \\ \sqrt{a_N a_0} & a_N \ll a_0 \end{cases} \quad \text{Empirical force law}$$

- Feedback
- MOND
- Modification of DM

Challenging to write a unified theory that describes those 3 phases.

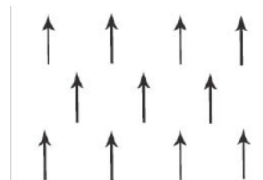
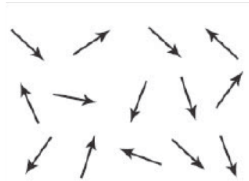
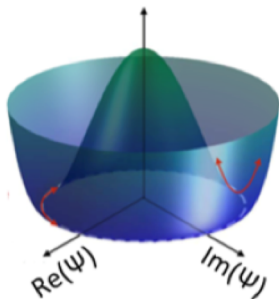
# Collective modes

- Described by the **collective** behaviour of the particles.
- No need for **microphysics**: symmetry alone describes the system.



- Collective modes related to symmetry = **Nambu–Goldstone bosons**

Collective modes from broken symmetries



Order parameter

Effective degree of freedom

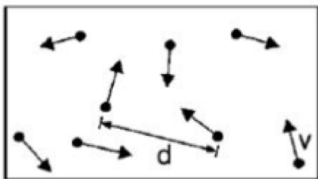


Effective dynamics

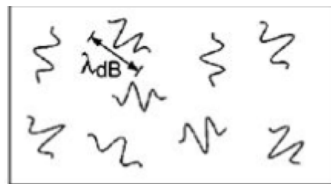
# Superfluid



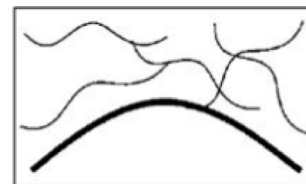
- At **low temperatures**, it is a superfluid when it **condensates** into a Bose-Einstein condensate (BEC).
- De Broglie wavelength ( $\lambda_B$ ) of each particle is large enough that their quantum wave function overlap, and a **single wave function** describes the **entire liquid**.
- Quantum phenomenon** that appears at low temperatures and macroscopic scales.
- Effective dynamics:** fluid that can flow **without friction**.



High temperature  
Thermal velocities

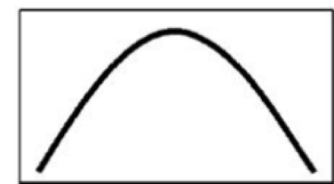


Low temperature  
 $\lambda_B \sim T^{-1/2}$   
"wave packets"



$T = T_c$   
BEC

"matter wave overlap"

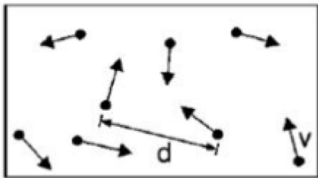


$T = 0$   
Pure BEC

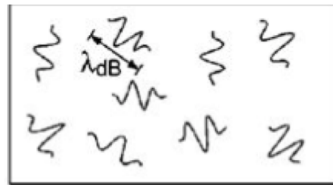
"giant matter w7ve" 6

## Goal:

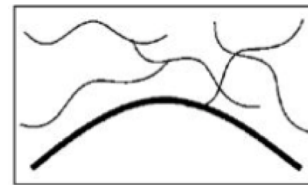
Use this property of the **collective** behaviour to explain the **modified dynamics** at certain **scales/times** in the evolution of the universe.



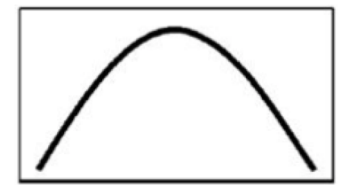
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$T = T_c$   
BEC  
"matter wave overlap"



$T = 0$   
Pure BEC  
"giant matter wave"<sup>8</sup>

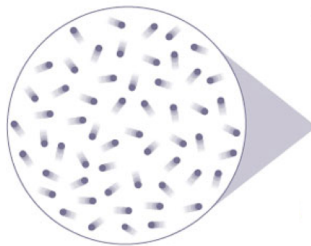
# Dark Matter Superfluid

Lasha, J. Khoury

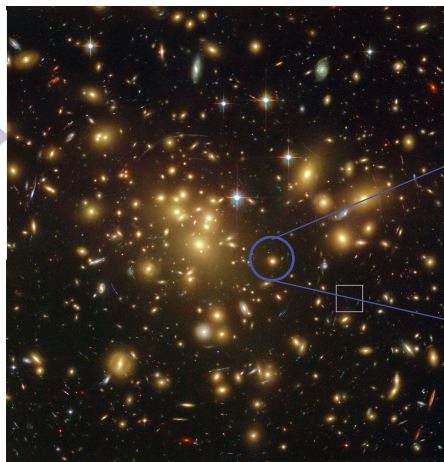
DM and MOND two phases of the same substance:

- Large scales: DM behaves like standard particle dark matter.
- Galactic scales: DM forms a condensate where collective macroscopic behavior leads to the modification of the dynamics at low accelerations.

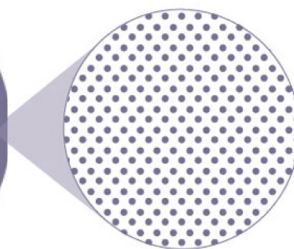
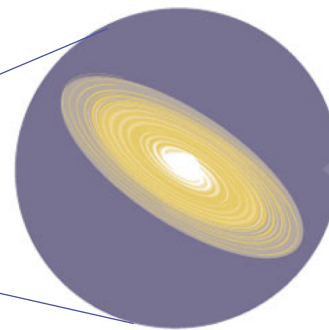
Large scales  
Clusters



DM: particles



Galaxy halo



DM: condensates

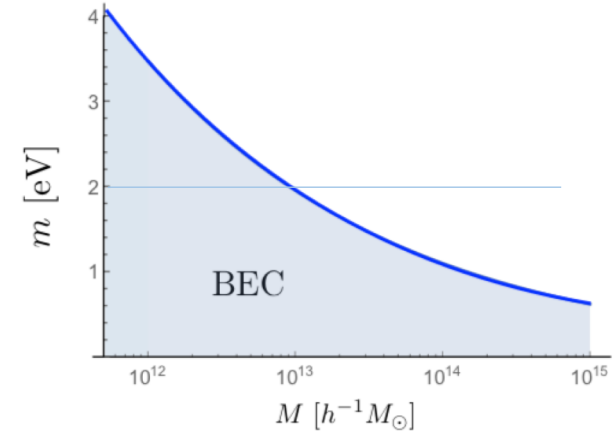
Adapted from Quanta

# Condition for Superfluidity

- DM has to condensate in galaxies: the de Broglie wavelength of the particles has to overlap.

$$\lambda_b \sim \frac{1}{mv} \gtrsim d \sim \left( \frac{m}{\rho_{vir}} \right)^{\frac{1}{3}}$$

$$\Rightarrow \boxed{m \lesssim 2 \text{ eV}}$$



Strongly interacting axion-like particle.  
DM is cold:  $T_c \sim \text{mK}$

Cold atoms in the lab

$$\frac{\sigma}{m} \gtrsim 0.1 \frac{\text{cm}^2}{g}$$



# Description of the superfluid

The present context of a bosonic superfluid is a **Bose Einstein condensate**, in the presence of **self interactions**.

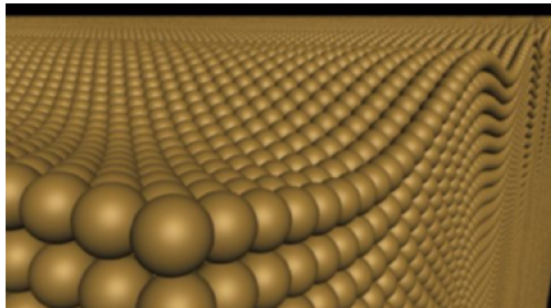
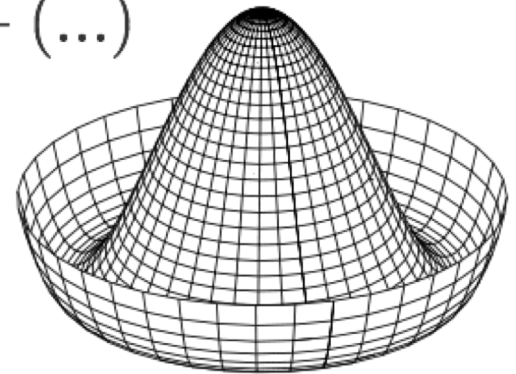
Effective field theory that describes a superfluid is represented by a:

- System with a **U(1) global symmetry** that is spontaneously broken.

$$\mathcal{L}_0 = -|\partial\Psi|^2 - m_\Psi^2|\Psi|^2 - \frac{\lambda}{2}\Psi^4 + (\dots)$$

Ansatz:  $\Psi_{bg} = \bar{v}e^{i\bar{\mu}t}$

Excitations:  $\Psi = (v + \rho) e^{i(\underbrace{\mu t + \theta}_{\theta})}$



**Collective excitations:** massless Goldstone and massive quasi-particles.

**Low energy:** only  $\theta$  excited - phonon

# Description of the superfluid

## Low energies ( $\dot{\theta}/m \ll 1$ )

Greiter, Wilczek & Witten (1989);  
Son and Wingate (2005)

- Low energy DOF: Only massless Goldstone bosons excited  $\theta$

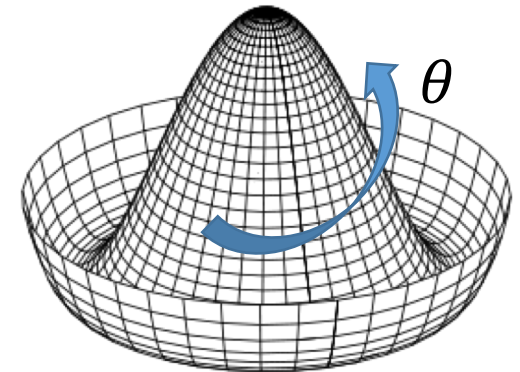
Shift symmetry  $\theta \rightarrow \theta + c$

$$\theta = \mu t + \pi$$

Chemical potential



Phonon excitations



$$c = \mu t$$

In the non-relativistic regime and at lowest order in derivatives:

$$\mathcal{L} = P(X), \quad X = \dot{\theta} - m\Phi - \frac{(\vec{\nabla})^2}{2m}, \quad \dot{\theta}/m \ll 1$$

Gravitational potential

# MOND from higher-derivative corrections

To describe non-relativistic DM and MOND-like:

$$\mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO/grav}} .$$

Leading order:

$$\mathcal{L}_{\text{LO}} = \frac{\Lambda^4}{n} \left( \frac{X}{m} \right)^n . \quad \left\{ \begin{array}{l} - n=2: P \sim \rho^2 \text{ - standard BEC.} \\ - n=3/2: P \sim \rho^3 \\ - n=5/2 \text{ (Unitary Fermi Gas): } P \sim \rho^{5/3} \end{array} \right.$$

**Next-to-leading order:** Correction involving  $(\vec{\nabla} X)^2 \rightarrow m^2 (\vec{\nabla} \Phi)^2$  modify the kinetic term for gravity.

Symmetry breaking potential:  $\mathcal{L}_{\text{NLO/grav}} = \mathcal{L}_{\text{NLO/grav}}(\vec{\nabla} \Phi)$

$$X = \dot{\theta} - m\Phi - (\vec{\nabla}\theta)^2/2m$$

---

OR by adding a coupling to baryons:  $\mathcal{L}_{\text{int}} \sim \frac{\Lambda}{M_{\text{Pl}}} \theta \rho_{\text{b}}$

# MOND from higher-derivative corrections

- Symmetry restoring:  $|\vec{\nabla}\Phi| > 3a_0$

- Symmetry breaking:  $|\vec{\nabla}\Phi| < 3a_0$

$$\mathcal{L}_{\text{NLO/grav}} \simeq -M_{\text{Pl}}^2 (\vec{\nabla}\Phi)^2$$

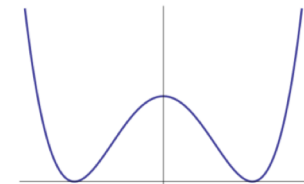
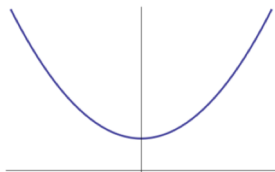
$$\mathcal{L}_{\text{NLO/grav}} \simeq -\frac{2M_{\text{Pl}}^2}{3a_0} \left( (\vec{\nabla}\Phi)^2 \right)^{3/2} + \frac{M_{\text{Pl}}^2}{9} \frac{(\vec{\nabla}\Phi)^4}{a_0^2}$$

$$\Rightarrow \boxed{\vec{\nabla}^2\Phi = \frac{\rho_s + \rho_b}{2M_{\text{Pl}}^2}}$$

$$\Rightarrow \boxed{\vec{\nabla} \cdot \left( \frac{|\vec{\nabla}\Phi|}{a_0} \vec{\nabla}\Phi \right) = \frac{\rho_s + \rho_b}{2M_{\text{Pl}}^2}}$$

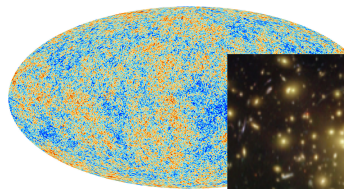
Newtonian limit

MOND limit



$$\chi = 0$$

$$\chi = \pm \sqrt{\frac{3a_0}{|\vec{\nabla}\Phi|} - 1}$$



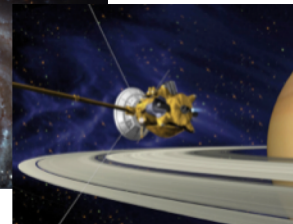
Only DM



Mostly DM



"MOND"



No MOND

# Description of the superfluid

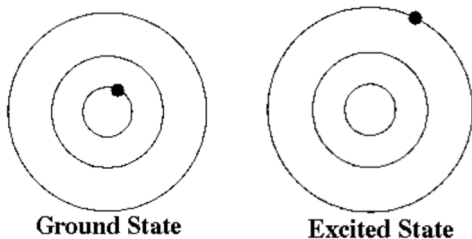
Low energy: phonon, vibration quanta- can be understood as sound waves.

But, not the entire story...

Taking a closer look into the superfluid: **atoms**.

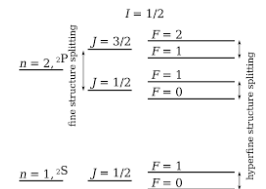
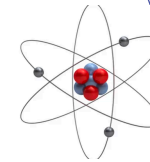
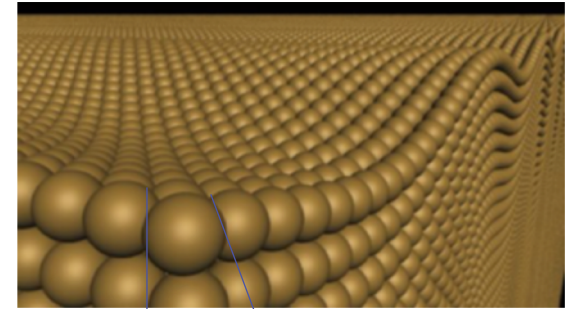
Full richness of the **internal DOF** needs to be taken into consideration. (Out of lab!)

Components of the (same) superfluid can be in **two distinguishable states**. (Refinement)



Atoms in these states are in contact, interact!

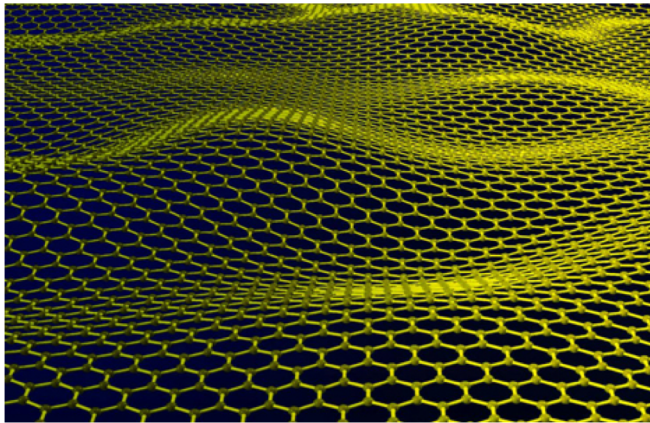
Phonon



# Description of the superfluid

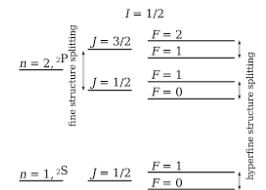
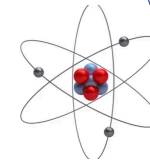
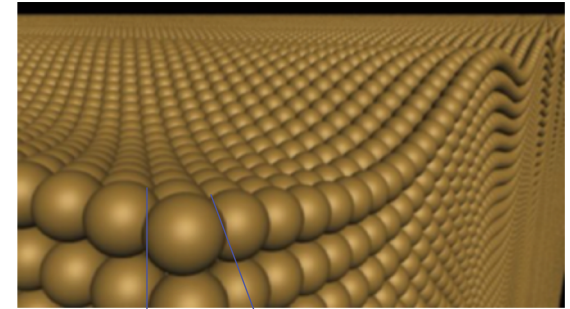
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Seol Seung-kwon (2014)

Phonon



Taking a closer look into the superfluid: **atoms**.

Superfluid with **two distinguishable states**.

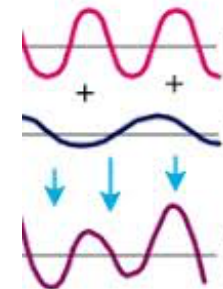


Phonons that propagate with **different phases** for **each species**



Phonons **interact!**  
The complex field interacts!

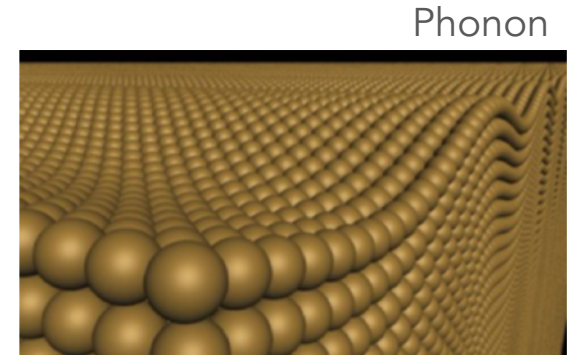
Analogous to interference of waves



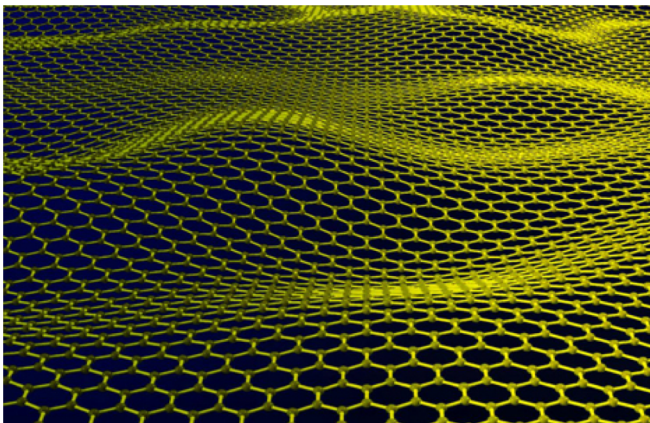


# Description of the superfluid

Low energy: phonon, vibration quanta- can be understood as sound waves.



But, not the entire story...



Seol Seung-kwon (2014)

Excitations in different parts of the superfluid come from **different configurations or states**



Phonons that propagate with **different phases at different locations**

E.g.: Superfluid described by the Cooper pairs

Different phases of the copper pairs in different parts of the superfluid describe distinct excitations.

# Description of the Superfluid

The present context of a bosonic superfluid is a Bose Einstein condensate, in the presence of self interactions.

Model by **two** superfluids:  $U(1) \times U(1)$  global symmetry

$$\Phi = (v + \rho)e^{i(\mu_1 t + \pi)}$$

$$\Psi = (v + \bar{\rho})e^{i(\mu_2 t + \bar{\pi})}$$

$$\mathcal{L}_0 = -|\partial\Phi|^2 - m_\Phi^2 |\Phi|^2 - \frac{\lambda}{2} |\Phi|^4 - |\partial\Psi|^2 - m_\Psi^2 |\Psi|^2 - \frac{\sigma}{2} |\Psi|^4,$$

Interaction: breaks  $U(1) \times U(1) \rightarrow U_r(1)$

$$\mathcal{L}_{int} = -\alpha \frac{\Phi^* \Psi + \Psi^* \Phi}{|\Phi| |\Psi|}$$

Josephson or Rabi coupling:

- Contact interaction;
- Long-range phase coherence b/ components;
- Conversion of species.

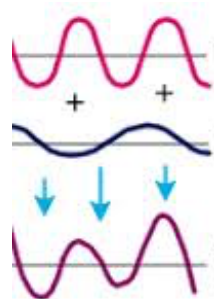
Low-energies:

$$\Theta_2 - \Theta_1$$

$$\mathcal{L} = P(X_1) + P(X_2) + 2\alpha \cos(\Theta_2 - \Theta_1)$$

Analogous to  
*interference of waves*  $\Rightarrow$

depends only on  
angle difference



Ref: P. Tommasini, E. J. V. de Passos, A. F. R. de Toledo Piza, and M. S. Hussein, PRA 67



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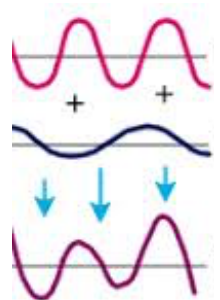
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$$\mathcal{L} = P(X) + 2\alpha \cos(\Theta_2 - \Theta_1)$$

- From condensed matter:
  - Josephson tunneling
  - Iron-based superconductors
  - MgB2
  - High Tc cuprate superconductor
  - XY model

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Low-energies:  $\Theta_2 - \Theta_1 = \theta_2 - \theta_1 + \Delta Et$

1: Ground state

2: Excited state



$$\mathcal{L} = P(X_1) + P(X_2) - 2\alpha \cos(\theta_2 - \theta_1 + \Delta Et)$$

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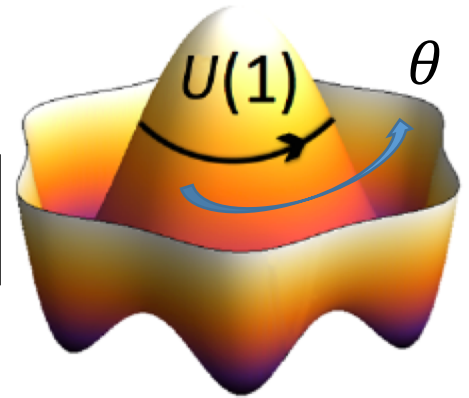
Well motivated to have a potential term!  
Can this potential drive accelerated expansion?

# Unified Dark Energy/Dark Matter Superfluid

## Low energy effective description

- Massless Goldstone bosons  $\theta_i$ , non-relativistic regime  $\Rightarrow \dot{\theta}/m \ll 1$

$$\begin{aligned} \mathcal{L} &= \sum P(X_i) - V(\theta_2 - \theta_1 + \Delta Et) \\ &= \sum \frac{\Lambda^4}{n} \left( \frac{X_i}{m_i} \right)^n - \frac{M^4}{2} \left[ 1 + \cos \left( \frac{\theta_2 - \theta_1 + \Delta Et}{f} \right) \right] \end{aligned}$$



- Oscillatory potential is well motivated from the fundamental description and phenomenologically:  $X = \dot{\theta} - m\Phi - (\vec{\nabla}\theta)^2/2m$

$\Rightarrow$  Breaks shift symmetry **weakly** into a **discrete** symmetry.

- Still **protects** the potential from **quantum corrections**.
- **Suppresses fifth forces**: naturally screened DE (and DM).

Analogous to a pNGB potential in  $\theta$ .

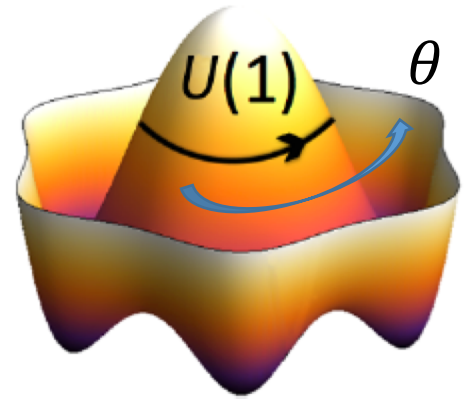
Frieman et al 1995

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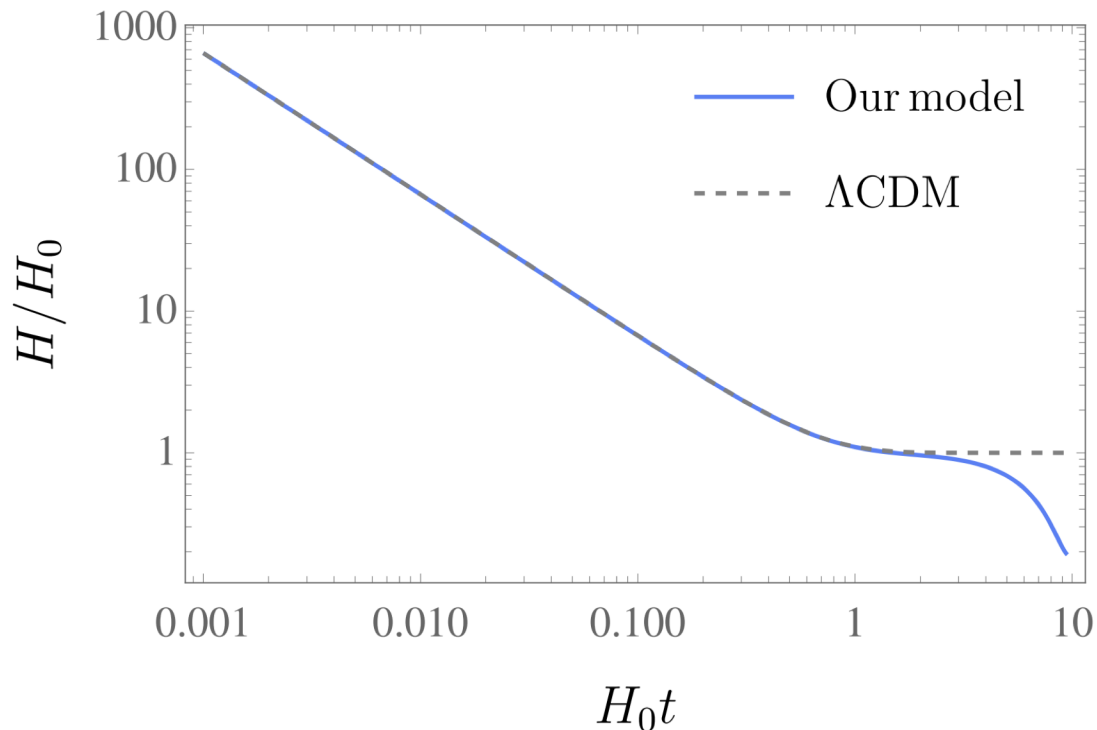
Valid for any  $P(X)$ !

$$H_{init} \ll \Delta E \ll m_i$$

# Background Evolution

In a FRW universe, for any  $P(X)$  superfluid in the NR limit:

$$2\dot{H} + 3H^2 = \frac{V(\Delta E t)}{M_{\text{Pl}}^2}$$



## Equation of state

$$w = \frac{p}{\rho} = \frac{\sum \frac{\Lambda^4}{n} \left( \frac{\dot{\theta}_i}{m_i} \right)^n - V(\Delta E t)}{\sum \frac{\Lambda^4}{n} \left( \frac{\dot{\theta}_i}{m_i} \right)^{n-1} + V(\Delta E t)}$$

For  $1 < n \leq 2$ :

- As  $\dot{\theta}$  decays  $\rightarrow$  Past: kinetic term dominates;  
Late times: potential term dominates.



## Equation of state

$$w = \frac{p}{\rho} = \frac{\frac{\Lambda^4}{n} \left(\frac{\dot{\theta}}{m}\right)^n}{\Lambda^4 \left(\frac{\dot{\theta}}{m}\right)^{n-1}} \quad \text{Past}$$

For  $1 < n \leq 2$ :

- As  $\dot{\theta}$  decays  $\rightarrow$  Past: kinetic term dominates;  
Late times: potential term dominates.
- Pressure has a higher power of  $\dot{\theta}/m$  than the energy density.  
So, in the regimes where the kinetic term dominates (past):

$$w \rightarrow \frac{\dot{\theta}}{m} \ll 1 \quad \rightarrow \quad \text{Dark matter!}$$

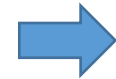
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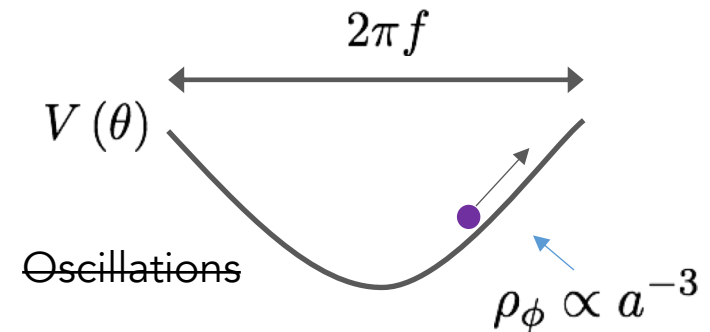
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$$w \rightarrow \frac{\dot{\theta}}{m} \ll 1$$



Dark matter!

- \* Naturally provides an EoS of matter- "Overdamped"
- \* No damped oscillations around the minimum of the potential, e.g., spintessence, pNGB, ...



\* It will always have small EoS and decaying, until potential term makes it negative.

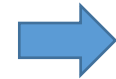
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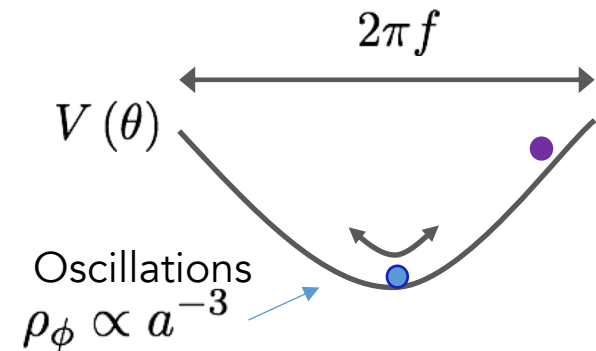
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For  $1 < n \leq 2$ :

- As  $\dot{\theta}$  decays, the kinetic term will be important in the past and the potential term at late times.
- Pressure has a higher power of  $\dot{\theta}/m$  than the energy density. So, in the regimes where the kinetic term dominates (past):

$$w \rightarrow \frac{\dot{\theta}}{m} \ll 1 \quad \rightarrow \quad \text{Dark matter!}$$

- As  $\dot{\theta}$  decays, when the potential term dominates we can have an EoS of acceleration, denoting the dark energy.

# Parameter Constraint

Free parameters:  $m, \Lambda, M$  and  $f$ .

Initial condition:  
matter-radiation equality

$$\rho_{eq} \simeq 0.4 \text{ eV}^4$$

$$c_{s,i} \ll 10^{-6}$$

$$n=2: \quad \Lambda_i = 500 \text{ eV}$$

Slow-roll: possible solution  
where  $V \gg K$ , for  $n=2$  :

$$f_\chi \sim 10^{-2} M_{pl}$$

Constraint

Final condition:  $H_0$   
constraints M

$$M \sim \text{meV}$$

Superfluid condition

$$m \sim \text{eV}$$

$$\Delta E \sim 10^{-11} \text{ eV}$$

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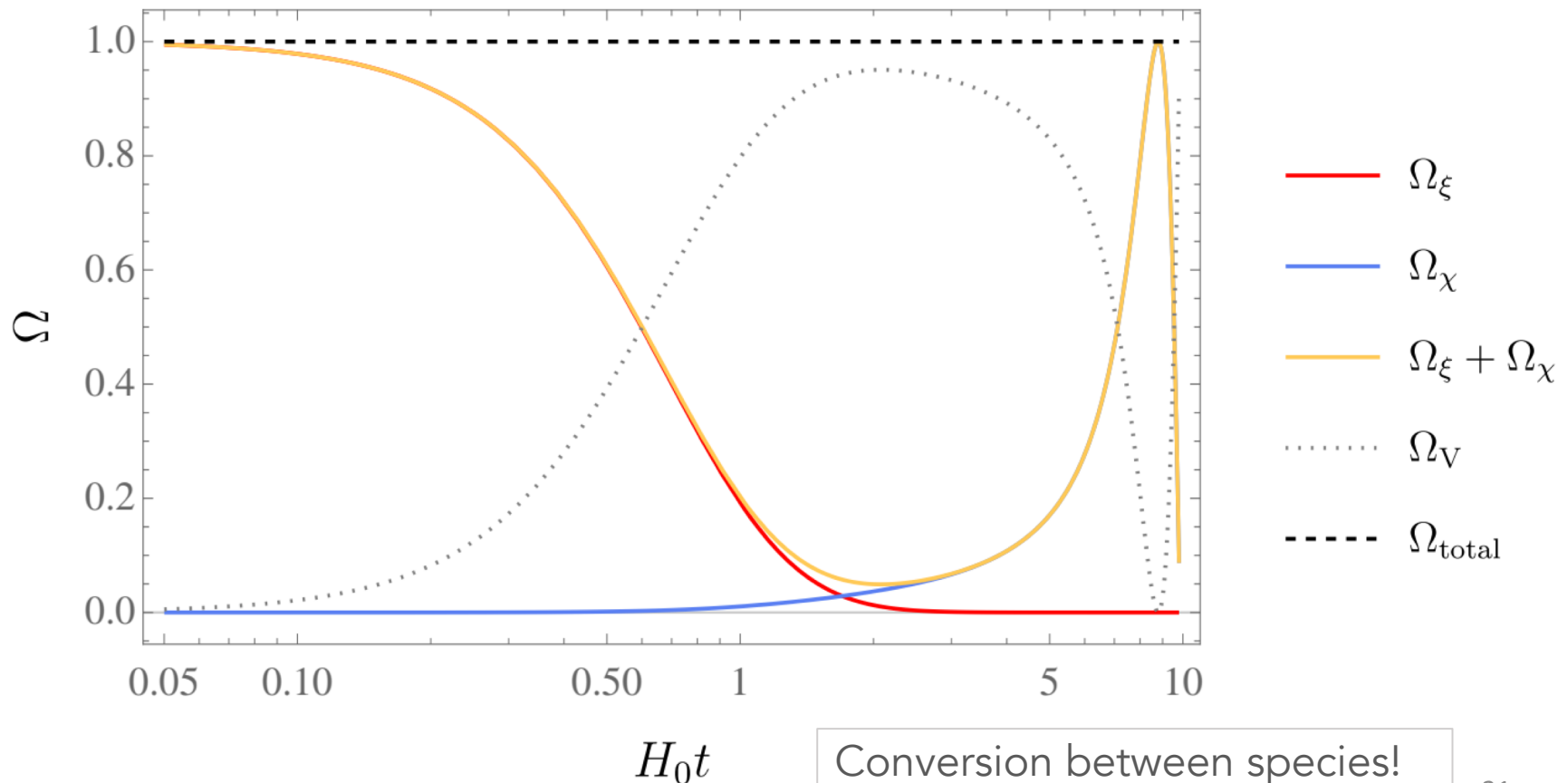
# Densities Evolution

Initial condition (eq): All DM in the ground state ( $\theta_1$ ) at eq.

Production mechanism: for sub-eV DM out of equilibrium

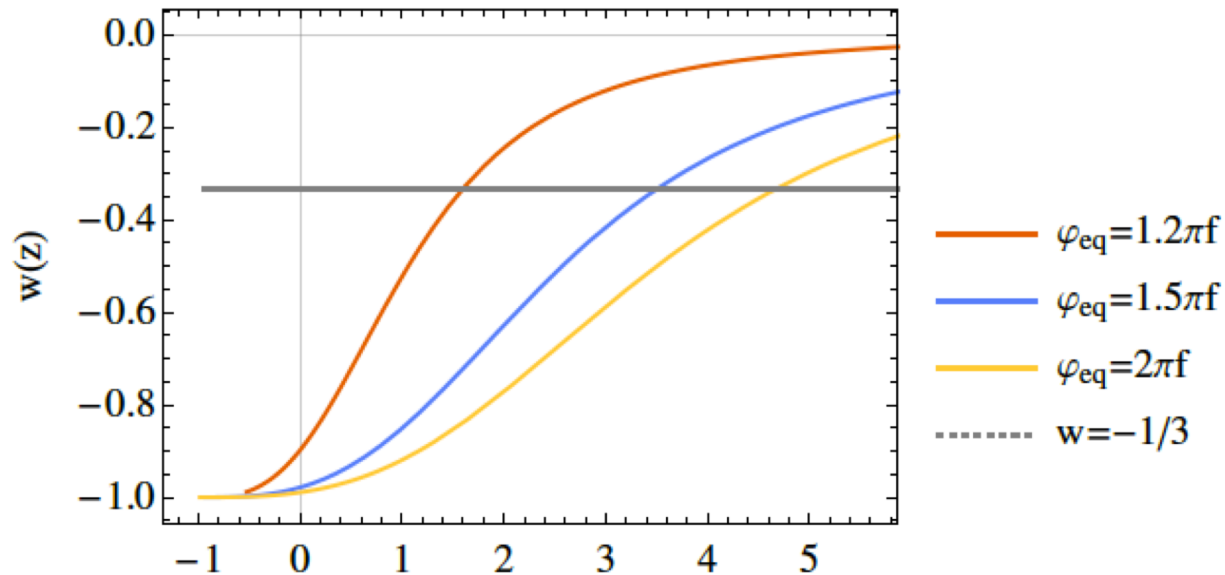
$$H \sim m = eV, z_{prod} \sim 10^{16} \Rightarrow T_{prod} \sim eV$$

$$\text{At eq.: } T_{eq}^{dm} \sim 10^{-26} \text{ eV} \Rightarrow T_{eq}^{dm} \ll \Delta E$$



## Equation of state

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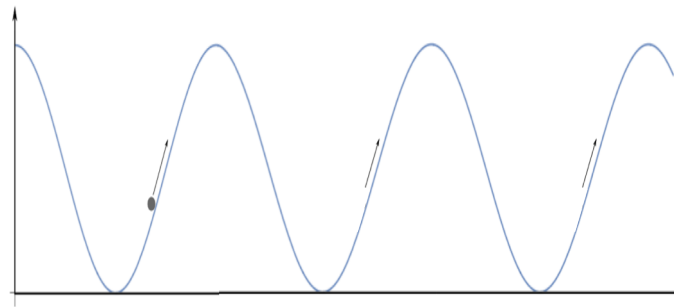
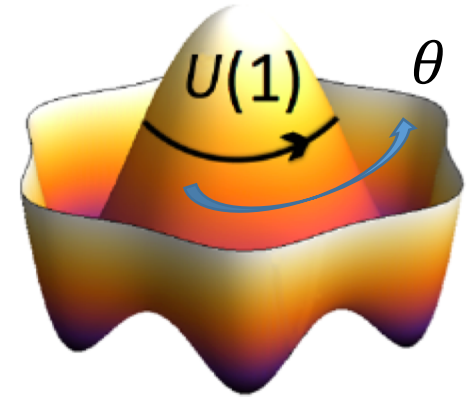
- Apparently it does not depend too much on the initial conditions: gets acceleration today – different instance to the coincidence problem. (Ongoing analysis)
- Exception: bottom of the potential  $(2n + 1)\pi f$  - takes too long to accelerate



# Cosmological Evolution

- **Radiation:**  $\dot{\theta}/m \sim 1$  (not described by the NR theory)

Field rotates fast for many cycles and does not feel the potential.



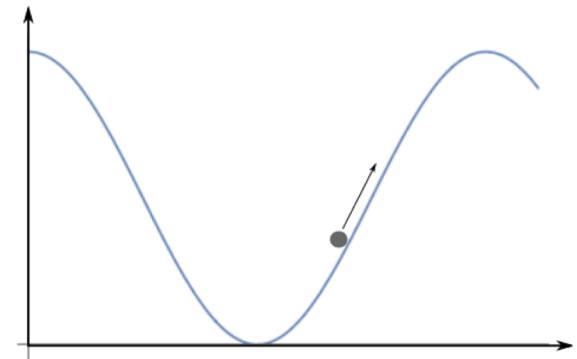
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$t_{eq}$

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Matter

$t_{dm/de}$

- **Dark Energy:**  $V(\theta)$  dominates.

Dark Energy

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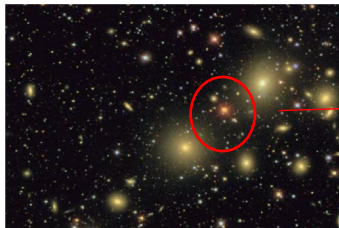
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$t_{dm/de}$

- **Dark Energy:**  $V(\theta)$  dominates.

Dark Energy



Small scales: gradients important  
DM behaviour on galactic scales



# Cosmological Evolution

FUTURE

- Dark Energy:  $V(\theta)$  dominates.

- Matter:  $\dot{\theta}/m \ll 1$ ;

Kinetic term dominates with  $w \rightarrow \frac{\dot{\theta}}{m} \ll 1$

- Dark Energy:  $V(\theta)$  dominates.



# Perturbations

Viable alternative to the  $\Lambda$ CDM :should be able to describe the growth of density perturbations that leads to the structures we observe.

Linear Newtonian perturbation theory  
Neglecting gradients

$$\ddot{\delta} + \left( 2H - \frac{\Delta E V'}{\bar{\rho}} \right) \dot{\delta} = \frac{1}{2M_{\text{Pl}}^2} \bar{\rho} \delta + \frac{\Delta E V'}{\bar{\rho}} \left( 5H + \frac{\Delta E V'}{\bar{\rho}} \right) \delta.$$

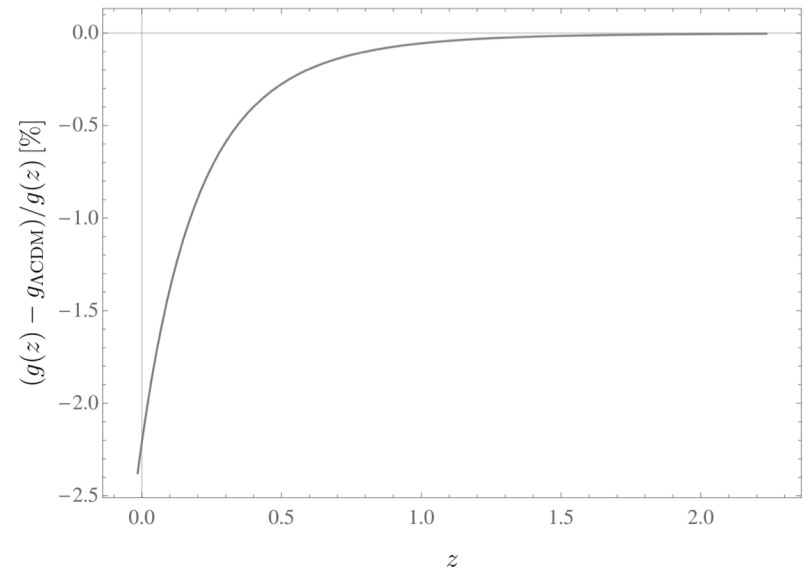
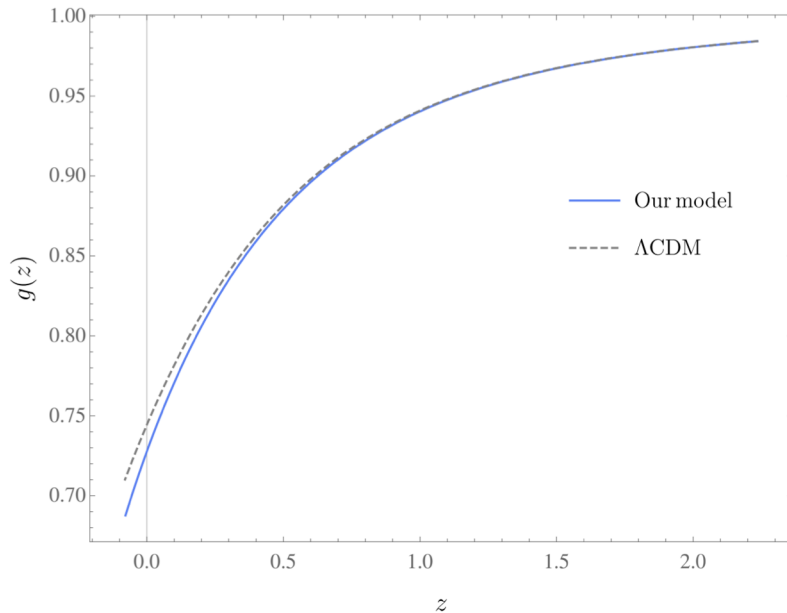
n=2:  $c_{s,i}^2 = \frac{\theta_i}{m} \ll 1$

# Observational Signatures

- Clustering: growth factor

Clustering in a model where the varying sound speed, dynamical evolution.

Growth function  $g(z) = \frac{1+z}{1+z_{\text{eq}}} \frac{\delta(z)}{\delta_{\text{eq}}}$



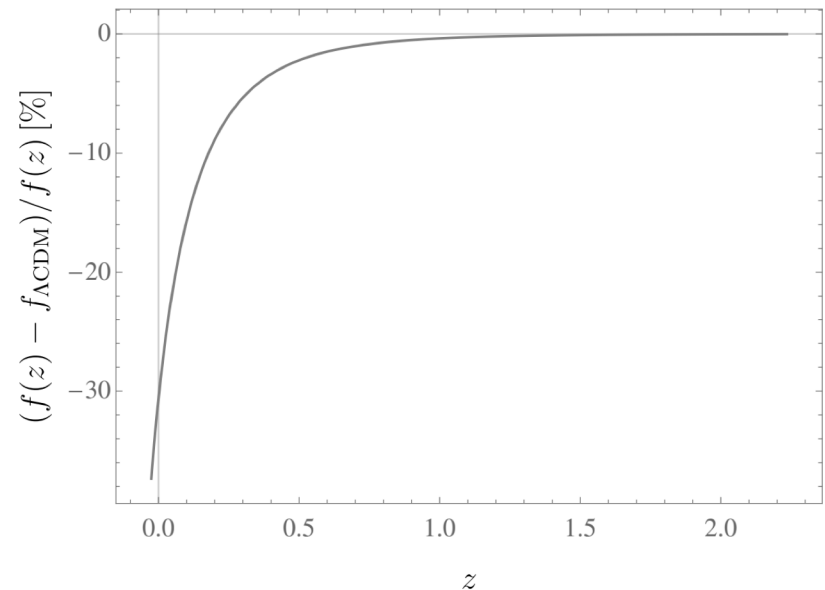
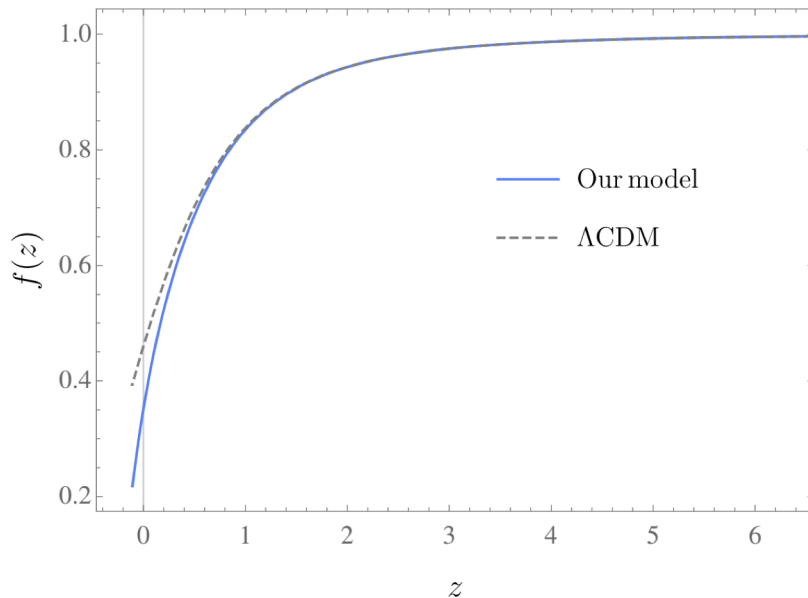
- Small suppression of density perturbation growth.
- > 2.0% difference

# Observational Signatures

- Clustering: growth factor

Clustering in a model where the varying sound speed, dynamical evolution.

Growth rate  $f(z) \equiv -\frac{d \ln \delta(z)}{d \ln(1+z)} = 1 - \frac{d \ln g(z)}{d \ln(1+z)}$



- Steeper suppression of density perturbation growth.
- > 30.0% difference from  $\Lambda$ CDM




# Summary

- New model that **unifies** the **large scale CDM**, **MOND** and **dark energy** in a single framework using the physics of superfluidity.
- **Only** needs the presence of **DM** in the form of a superfluid.
- The theory is found in analogue **condensed matter system** that motivates the origin and the choice of potential.
- Observational signatures:  
Predicts: 30% deviation in the growth rate from  $\Lambda$ CDM.

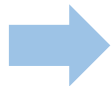
# Future

- Local DE contribution

Soon after production, DM becomes superfluid

$$\left(\frac{T}{T_c}\right)_{\text{cosmo}} \simeq 10^{-28} \quad \text{vs} \quad \left(\frac{T}{T_c}\right)_{\text{MW}} \simeq 10^{-2}$$


Difference  
abundance of  
ground and  
excited states



Different local acceleration in galaxies?

- Hydrodynamical simulations (Illustris – Prof. Volker Springel) – solve Gross-Pitaevskii equation.
- Observational signatures
  - Clustering, halo abundance and cluster counts
  - Non-linear regime
  - Effects from the new description of DM: vortices; ...
- Dynamical M;

Thank you!

# Phenomenological consequences of SfDM

Sf phonon coupled to baryonic matter

[Lasha Berezhiani](#), [Benoit Famaey](#), [Justin Khoury](#), 1711.05748

- Rotation curves of both high and low surface brightness galaxies can be reproduced
  - Slightly rising rotation curve at large radii in massive high surface brightness galaxies -> subtly different from Milgrom's law
- Expected differences with Milgrom's law
  - Dwarf spheroidal satellite galaxies, tidal dwarf galaxies, and globular clusters -> Milgromian or Newtonian behavior depends on the position with respect to the superfluid core of the host galaxy.
  - Ultra-diffuse galaxies within galaxy clusters to have velocities slightly above the BTFR.
- Photons and gravitons follow the same geodesics, and that galaxy-galaxy lensing, probing larger distances within galaxy halos than rotation curves, should follow predictions closer to the standard cosmological model than those of Milgrom's law.

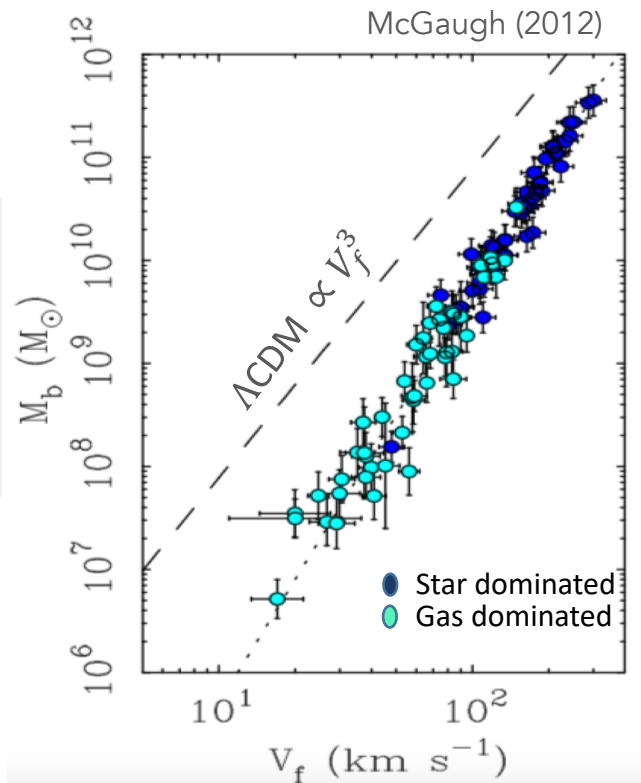
# Small Scale Challenges

## Galaxies

- Baryonic Tully Fisher Relation (BTFR)

Remarkably **tight** scaling relations between dynamical and baryonic properties.

Baryonic Mass

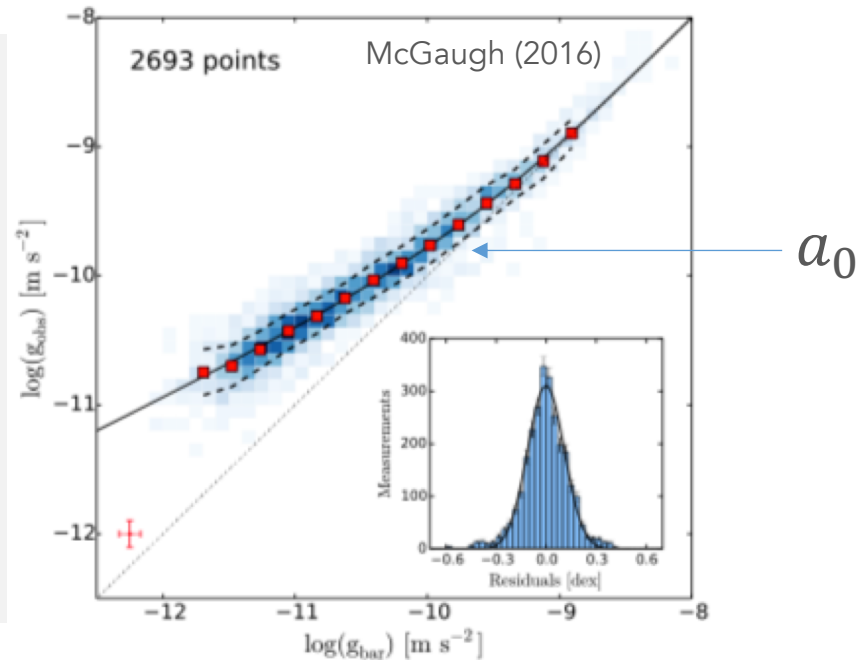


Circular velocity

$$a_0 \simeq \frac{1}{6} H_0 \simeq 1.2 \times 10^{-8} \text{ cm/s}^2 = 2.7 \times 10^{-34} \text{ eV}.$$

- Radial acceleration relation (RAR)

Radial acceleration by rotation curves  
( $g_{\text{obs}} = V^2/r$ )



Predicted radial acceleration from baryons

# MOND from higher-derivative corrections

To describe non-relativistic DM and MOND-like:

$$\mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO/grav}} .$$

Leading order:

$$\mathcal{L}_{\text{LO}} = \frac{\Lambda^4}{n} \left( \frac{X}{m} \right)^n . \quad \left\{ \begin{array}{l} - n=2: P \sim \rho^2 \text{ - standard BEC.} \\ - n=3/2: P \sim \rho^3 \\ - n=5/2 \text{ (Unitary Fermi Gas): } P \sim \rho^{5/3} \end{array} \right.$$

**Next-to-leading order:** Correction involving  $(\vec{\nabla} X)^2 \rightarrow m^2 (\vec{\nabla} \Phi)^2$  modify the kinetic term for gravity.

$$\mathcal{L}_{\text{NLO/grav}} = -\frac{1}{2} Z^2 (\partial \chi)^2 - M_{\text{Pl}}^2 (\vec{\nabla} \Phi)^2 \left( \frac{1}{1 + \chi^2} + \frac{(\vec{\nabla} X)^2}{9m^2 a_0^2} \chi^2 \right) .$$

Symmetry is broken or not depending on the sign of the mass at  $\chi = 0$

$$m_\chi^2 = \left. \frac{d^2 V}{d\chi^2} \right|_{\chi=0} = 2M_{\text{Pl}}^2 (\vec{\nabla} \Phi)^2 \left( -1 + \frac{(\vec{\nabla} \Phi)^2}{9a_0^2} \right)$$

# Dark Matter Superfluid: Cosmology

Axion-like  
 $m \sim \text{eV}$



- Must be produced out of equilibrium.
- Decoupled from ordinary matter throughout the cosmological history.
- Self-interactions: reaches thermal equilibrium at  $T \ll T_{\gamma b} \Rightarrow$  superfluid!
- Cosmologically it remains superfluid forever after.

Relativist Completion

$$V(\chi) = -M_{\text{Pl}}^2 \left\{ \frac{R}{2} - \left( \frac{2}{3} \square n + \frac{\square \mathcal{Y}}{m^2} \right) \right\} F \quad \begin{cases} \Theta = mt + \theta \\ \mathcal{Y} = -\frac{1}{2}(\partial\Theta)^2 \simeq \frac{m^2}{2} + mX \end{cases}$$



$$m_\chi^2 = 3H^2 M_{\text{Pl}}^2 \left( 1 + \frac{\dot{\mathcal{Y}}^2}{9m^4 a_0^2} \right)$$

Always positive!

Cosmologically: always in the Einstein-gravity, symmetry-restoring phase



Background expansion history and linear growth of perturbations indistinguishable from  $\Lambda$ CDM.

# Long Range Forces

Dangerous terms like

$$\mathcal{L} \propto M \bar{\psi}\psi$$

are forbidden by the (discrete) shift symmetry.

Allowed terms

$$\delta\mathcal{L} \sim \beta' \frac{\partial_\mu \phi}{M_P} \bar{\psi} \gamma^\mu \gamma^5 \psi$$

$$\mathcal{L} \supset \sum_{i=1}^N \alpha \frac{\phi_i}{4 M_P} F_{\mu\nu} \tilde{F}^{\mu\nu}$$



# Numerical Analysis

- Phantom Dark Energy

$$f = 10^{33}, \quad \Lambda \sim 5\text{eV}$$

Equation of state

