


Observations on the Development of Quantum Field Theory in Curved Spacetime



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- “A theory is the more impressive the greater the simplicity of its premises is, the more different kinds of things it relates, and the more extended is its area of applicability. Therefore the deep impression which classical thermodynamics made upon me. It is the only physical theory of universal content concerning which I am convinced that, within the framework of applicability of its basic concepts, it will never be overthrown.”

--Albert Einstein

What is QFT in CST?

- Valid at all energies.

$$Z = \int \mathcal{D}\chi \, e^{iS[\chi]}$$

- Quantized nonlinear gravity and particles. QG

$$Z \simeq \int \mathcal{D}g \, \mathcal{D}\phi \, e^{iS[g, \phi]}$$

- Quantized linear metric perturbations and particle fields on background metric g , valid well below the Planck scale. This is QFT in CST

$$Z \simeq \int \mathcal{D}\phi \, e^{iS[g, \phi]}$$

1. Particle fields are quantized and obey field equations, such as:

$$(\square + m^2) \varphi = 0$$

2. The background metric g typically satisfies the semi-classical Einstein equation:

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + \Lambda g^{\mu\nu} = 8 \pi G [(T^{\mu\nu})_{\text{classical}} + \langle T^{\mu\nu} \rangle]$$

The expectation value on the right of (2) gives a good approximation to the energy distribution of the field for states having sufficient symmetry, as in a FRW expanding universe.

Is QFT in CST Important?

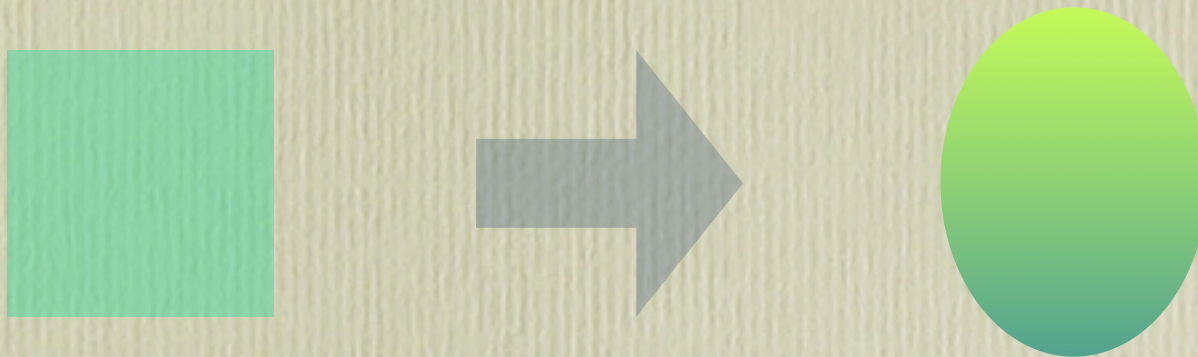
- It gives definite predictions that turn out to be fundamental and, in some cases, observable. For example:
- Quantized perturbations are amplified by the rapid expansion of the inflationary universe, ultimately giving rise to the anisotropies of the CMB observed today.
- Black holes emit thermal radiation and have a temperature and entropy consistent with the generalized second law of thermodynamics.

Hidden Treasure

- Most of the effort in trying to combine QFT and GR before 1962 was devoted to the fundamental problem of quantizing the gravitational field to produce a theory of quantum gravity (QG). Bergmann, Dirac, Schwinger, DeWitt, Deser, Wheeler and others devoted much effort to QG.
- Some work on QFT in CST was done by Moller, Utiyama, DeWitt, Imamura and others, but the treasures remained hidden.

As a graduate student, I was determined to work at the interface of general relativity and quantum field theory. In 1962, my Ph.D. thesis advisor, the great particle physicist Sidney Coleman at Harvard, suggested that I look at the question of whether any particles were created by the expanding universe. Totally unaware of any previous work on QFT in CST, I chose a careful deductive approach that uncovered results and insights that proved useful to many researchers and led to fundamental advances.

Propagate the local properties of quantum field theory from flat into curved spacetime



$$(\square + m^2) \varphi = 0$$

For a general expanding universe, I found:

- Particle number is an adiabatic invariant
- Particles are created from vacuum
 - Growth of perturbations in cosmology
- Particle number and vacuum state become fuzzy
- Adiabatic regularization gives particle number operator measured by an idealized apparatus
- Spin-Statistics from curved space evolution

- Massless particles obeying conformally invariant field equations are not created in an arbitrary smooth isotropic expansion of the universe
- For those fields in such a universe, particle number and vacuum state are well defined
- Photons are an example, as are conformally-invariant gravitons, but not gravitons obeying the linearized Einstein equation - the latter are like minimally coupled scalar fields that I showed create particles.
- Here are some illustrations of what happens:

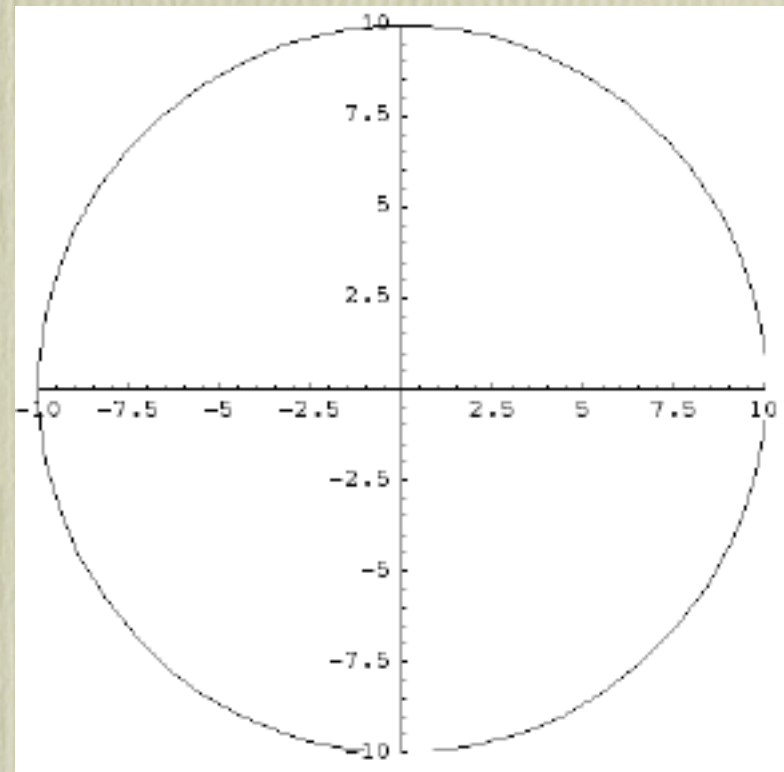
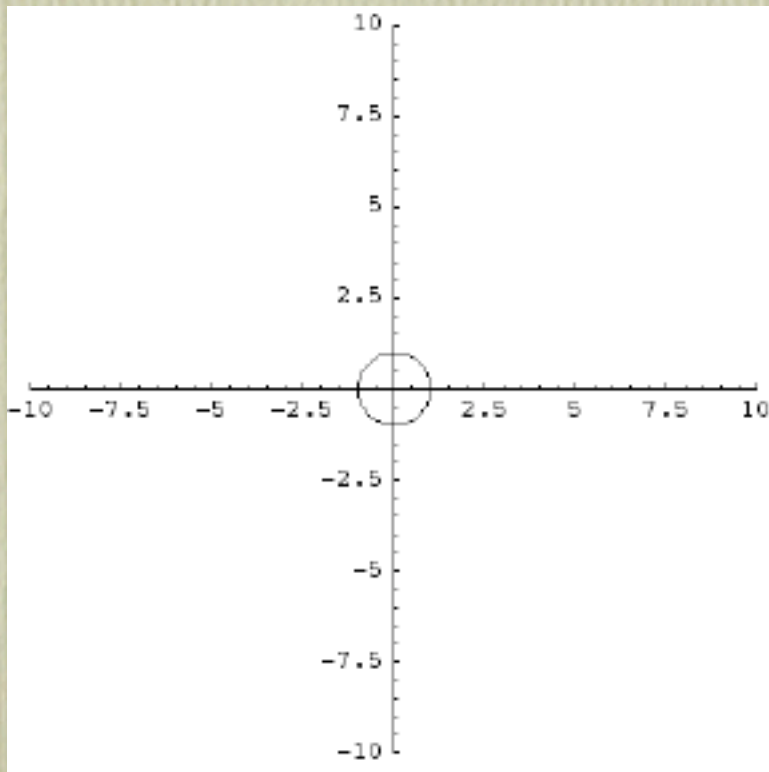
Evolution mixes annihilation and creation operators in a Bogolubov transformation. Initial vacuum has particles at late times, so particles are created by the expansion.

- Early-time annihilation op

$$a_{\vec{k}}$$

- Late-time annihilation op

$$A_{\vec{k}} = \alpha_k a_{\vec{k}} + \beta_k a_{-\vec{k}}^\dagger$$



Static universe expands and becomes static again

Implies Particle Creation from Vacuum

- Pairs created from vacuum

$$\langle N \rangle = \sum_{\vec{k}} |\beta_k|^2$$

- Stimulated pair creation when particles present

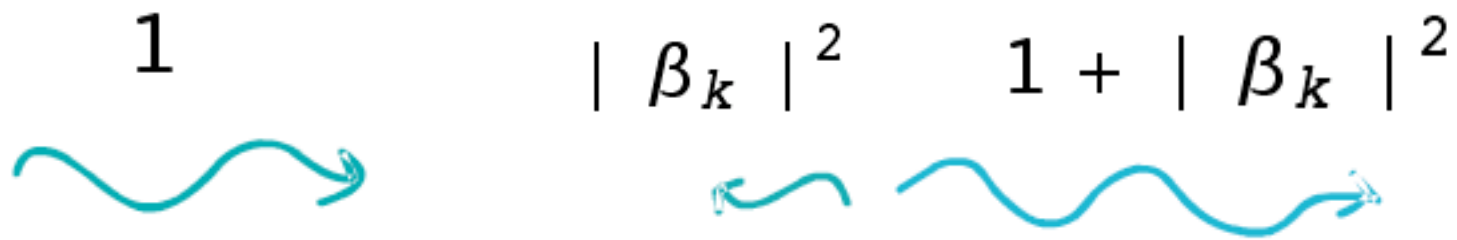
$$\langle N \rangle = \langle N_0 \rangle + \sum_{\vec{k}} |\beta_k|^2 (1 + 2 \langle N_{\vec{k}0} \rangle)$$

$$(\square + m^2) \varphi = 0$$



$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

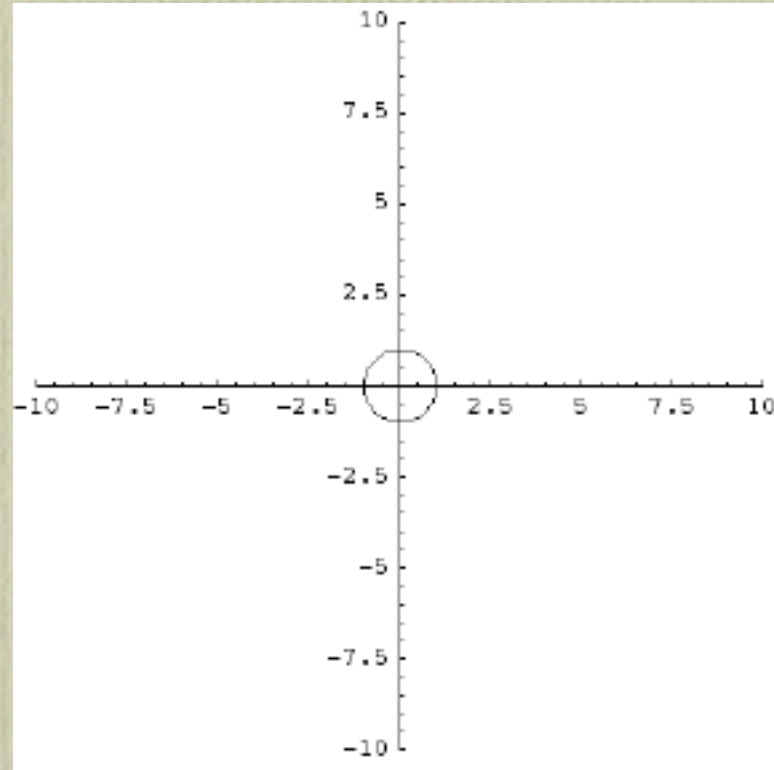
Amplifies Fluctuations During Inflation



- Origin of fluctuations of 1 part in 10^5 now observed in the cosmic background radiation

Spin-Statistics Proof start

Before Expansion of Universe

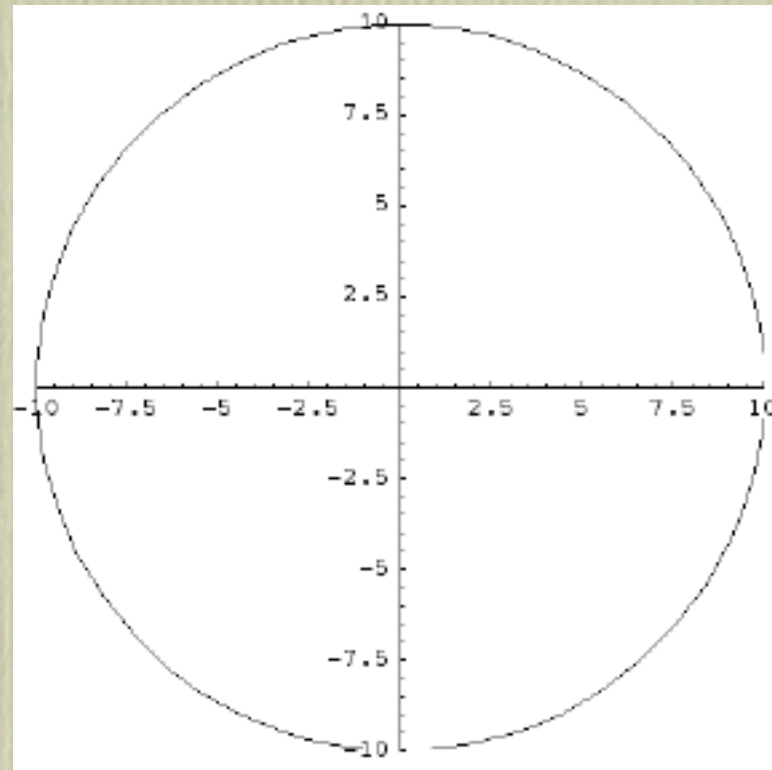


$$[a_{\vec{k}}, a_{\vec{k}'}^\dagger] = \delta_{\vec{k}, \vec{k}'}$$

Early time commutator

Spin-Statistics Proof finish

After Expansion of Universe



$$[A_{\vec{k}}, A^{\dagger}_{\vec{k}'}] = (|\alpha_k|^2 - |\beta_k|^2) \delta_{\vec{k}, \vec{k}'}$$

Late-time commutator: new factor is conserved Wronskian = 1

- For integer spin recall that:

$$(\square + m^2) \varphi = 0 \quad \rightarrow$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

- For half-odd integer spin, one finds that:

$$(\mathbf{i}\gamma^\mu \nabla_\mu - m) \psi = 0 \quad \rightarrow$$

$$|\alpha_k|^2 + |\beta_k|^2 = 1$$

- Then only commutators evolve properly for integer spin:

$$[a_{\vec{k}}, a_{\vec{k}'}^\dagger] = [A_{\vec{k}}, A_{\vec{k}'}^\dagger]$$

- And only anti-commutators evolve properly for half-odd integer spin:

$$\{a_{\vec{k}}, a_{\vec{k}'}^\dagger\} = \{A_{\vec{k}}, A_{\vec{k}'}^\dagger\}$$

Black Holes Create Particles

historical notes

- As early as 1965, I realized that a body collapsing to form a black hole creates particles. I proposed this project in my first NSF proposal in 1968.
- In 1971 I spent a year at Princeton in Wheeler's group and was invited by Wightman to be second reader on Fulling's thesis.
- Fulling showed that in Rindler coordinates formed by a set of accelerating observers in Minkowski space, the Minkowski creation and annihilation operators are mixed by a non-diagonal Bogolubov transformation, so Rindler particles are present in the Minkowski vacuum.

- Later Davies and Unruh realized that the spectrum of particles Fulling had found was black-body, and Unruh showed that an accelerated particle detector would actually respond to those particles. More recently, Matsas and Vanzella extended these ideas to analyze the decay of accelerated protons.
- Before that, in 1973, Fulling became my first postdoctoral fellow and we set out to see if the close relation between Rindler coordinates in flat spacetime and Kruskal coordinates implies that a Schwarzschild black hole creates particles.

- But before we could complete our work on this project, Hawking published his great paper on exploding black holes in 1974
- Hawking showed that a body collapsing to form a black hole emits a flux of outgoing thermal radiation of temperature inversely proportional to the mass of the black hole. As the black hole evaporates through this radiation, its temperature increases and it ultimately explodes. Sufficiently light black holes from the early universe might be exploding today!

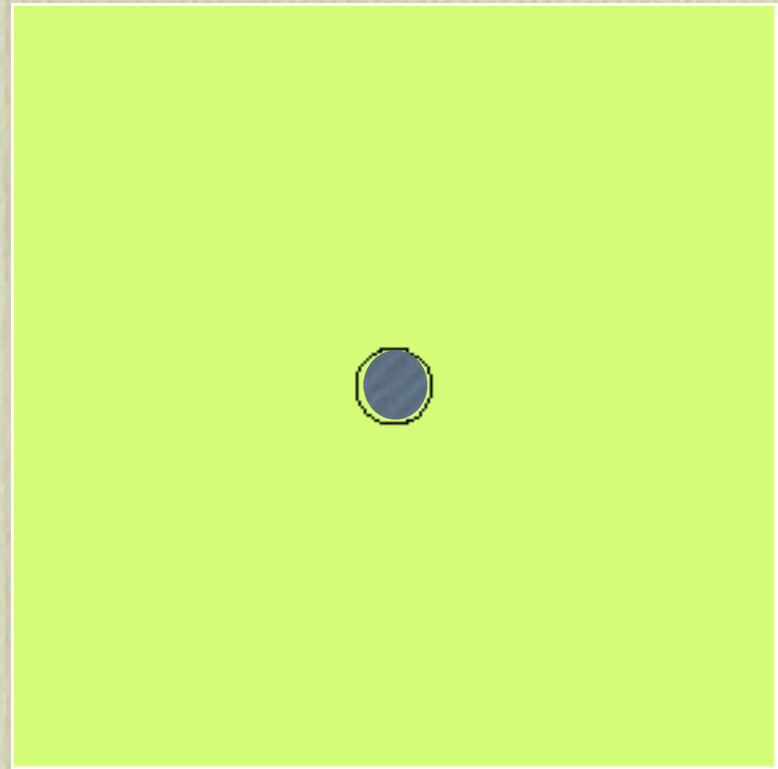
Collapse to a Black Hole

Early-time annihilation operator

$$a_j$$

Late-time annihilation operator

$$A_j = \sum_{j'} (\alpha_{jj'} a_{j'} + \beta_{jj'} a_{j'}^\dagger)$$



Early-time vacuum has particles at late times.
This particle creation is the Hawking radiation

- Radiation from the black hole is thermal

- Temperature: $T_H = \frac{\hbar c^3}{8 \pi k_B G M}$

- Entropy: $S = \frac{1}{4} k_B \left(\frac{c^3}{G \hbar} \right) \text{Area}$ • $\text{Area} = 4 \pi \left(\frac{2 G M}{c^2} \right)^2$

- Black hole of mass $M < 10^{14}$ gm evaporates in less than the age of the universe. Are correlations between what went in and what comes out lost during evaporation, or does the information stored in the correlations leak out?
- Microscopic explanation of the entropy: Strings?

Back to Thermodynamics

- The most important reason to be confident of Hawking's result was that it confirmed the interpretation of the generalized second law of thermodynamics that Bekenstein had postulated for systems that include black holes
- GR and QFT must be connected to enforce the second law of thermodynamics
- Strominger and Vafa, and others found an explanation of the entropy of supersymmetric BHs based on the microstates of strings.

- With thermodynamics as a guide, we see that GR and QFT in CST are leading us toward a deeper level of the underlying laws governing the universe. We are still near the beginning of a long road ahead.
- Now I would like to change topics, and take a few moments to comment on recent research in QFT in CST that I have been doing with Caldwell, Komp, and Vanzella, based on earlier work done with Raval.

Accelerating the Recent Expansion of the Universe

- Suppose that the underlying fundamental theory (strings?) gives a simple low energy limit with a field obeying the simple free field equation we have been considering: $(\square + m^2) \varphi = 0$
- Obtain the vacuum expectation value of the energy-momentum tensor of this field from the effective action W . The latter is defined by doing a Feynman functional integration over fluctuations of the field in the state in which its expectation value vanishes.

- Effective action:

$$e^{iW} = \int \mathcal{D}\phi e^{iS}$$

- Energy-momentum tensor:

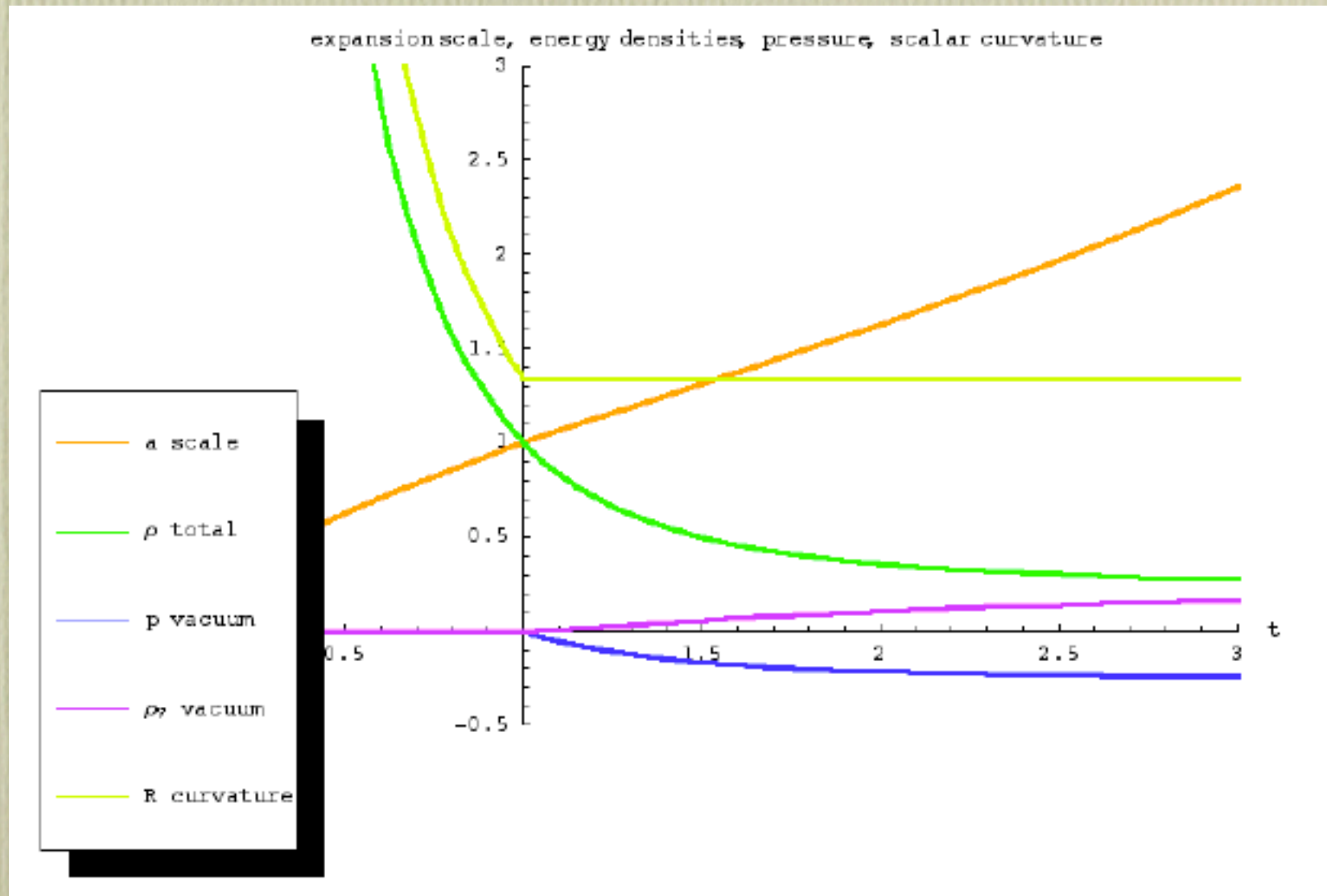
$$\langle T^{\mu\nu} \rangle = \frac{2}{\sqrt{-g}} \frac{\delta W}{\delta g_{\mu\nu}}$$

- Einstein gravitational field equation:

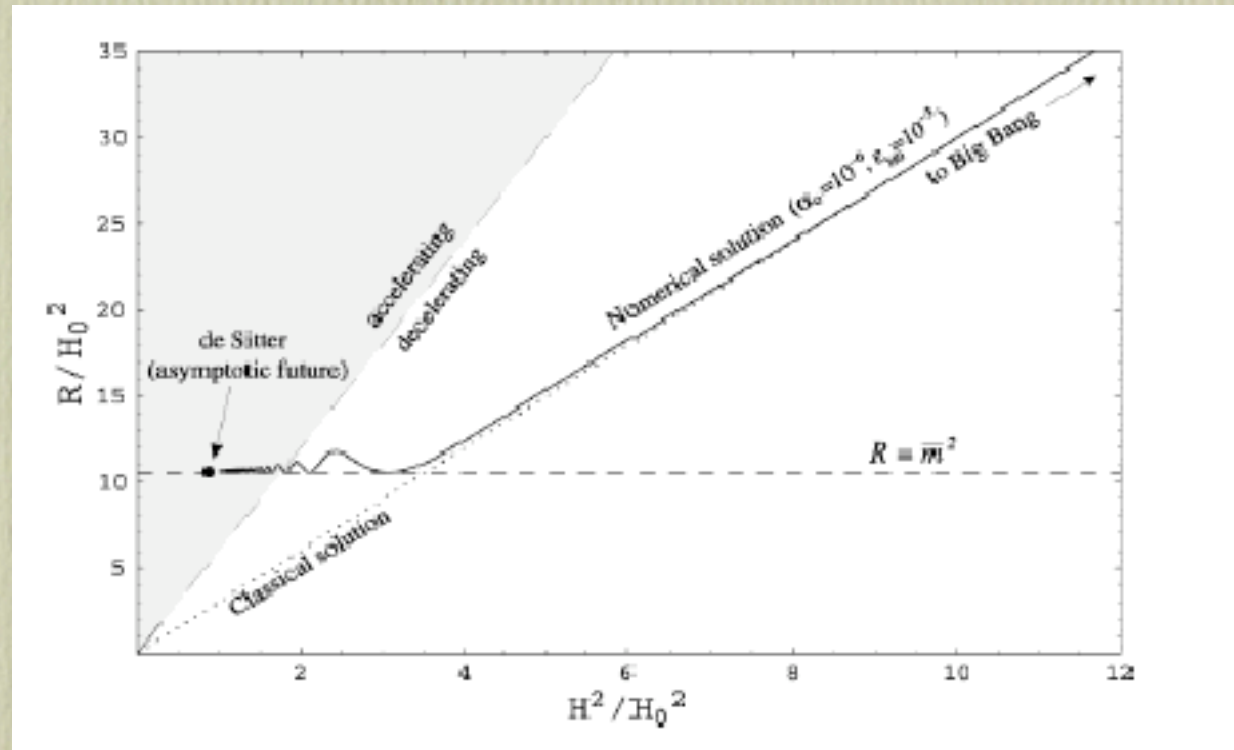
$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + \Lambda g^{\mu\nu} = 8\pi G [(T^{\mu\nu})_{\text{classical}} + \langle T^{\mu\nu} \rangle]$$

- Set the cosmological constant to **zero** and treat the cold-dark-matter and radiation classically. Numerically integrate this “vacuum cold-dark-matter model” (VCDM) for an isotropic expanding universe.

Results: Graphical Summary

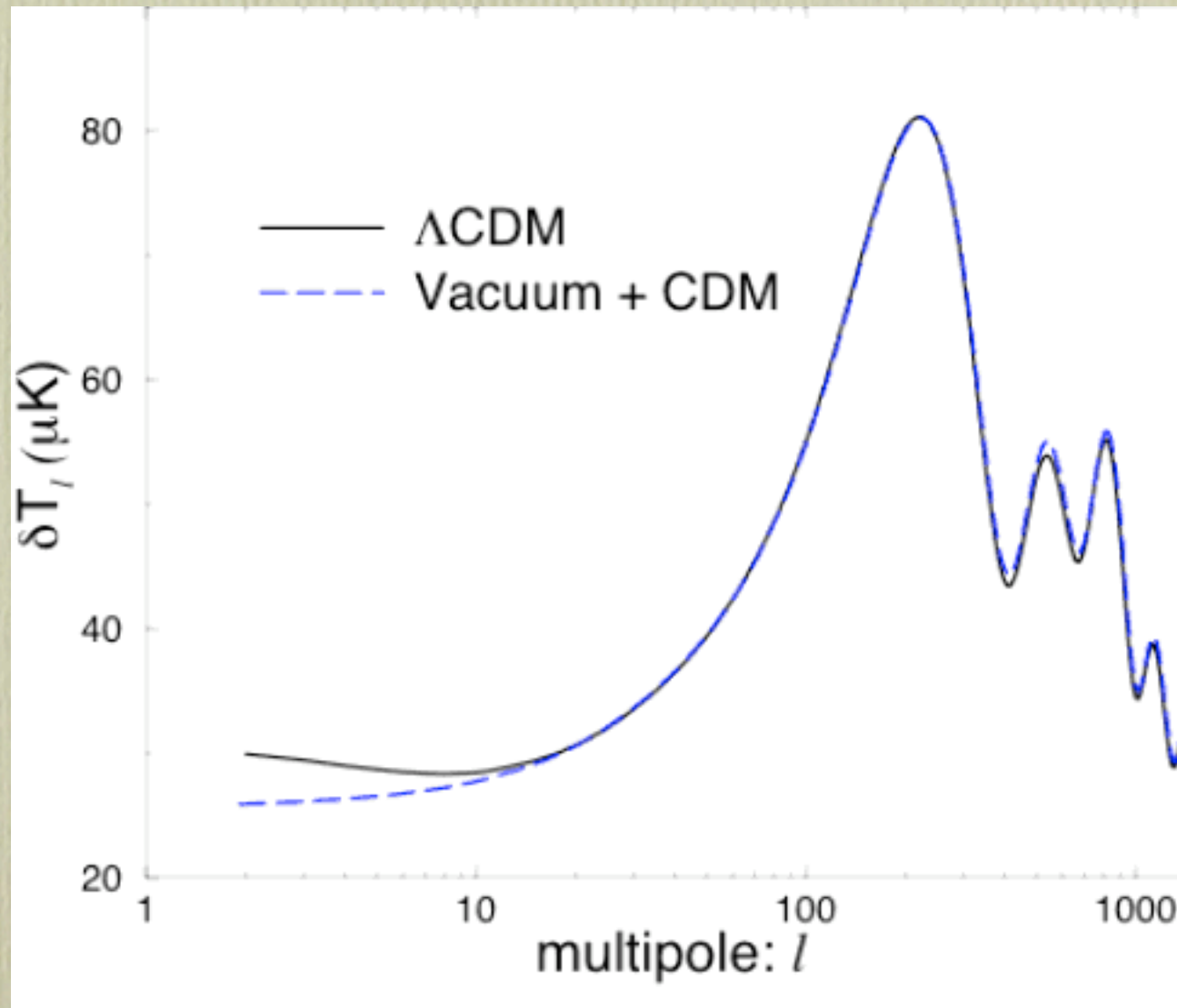


Numerical Solution

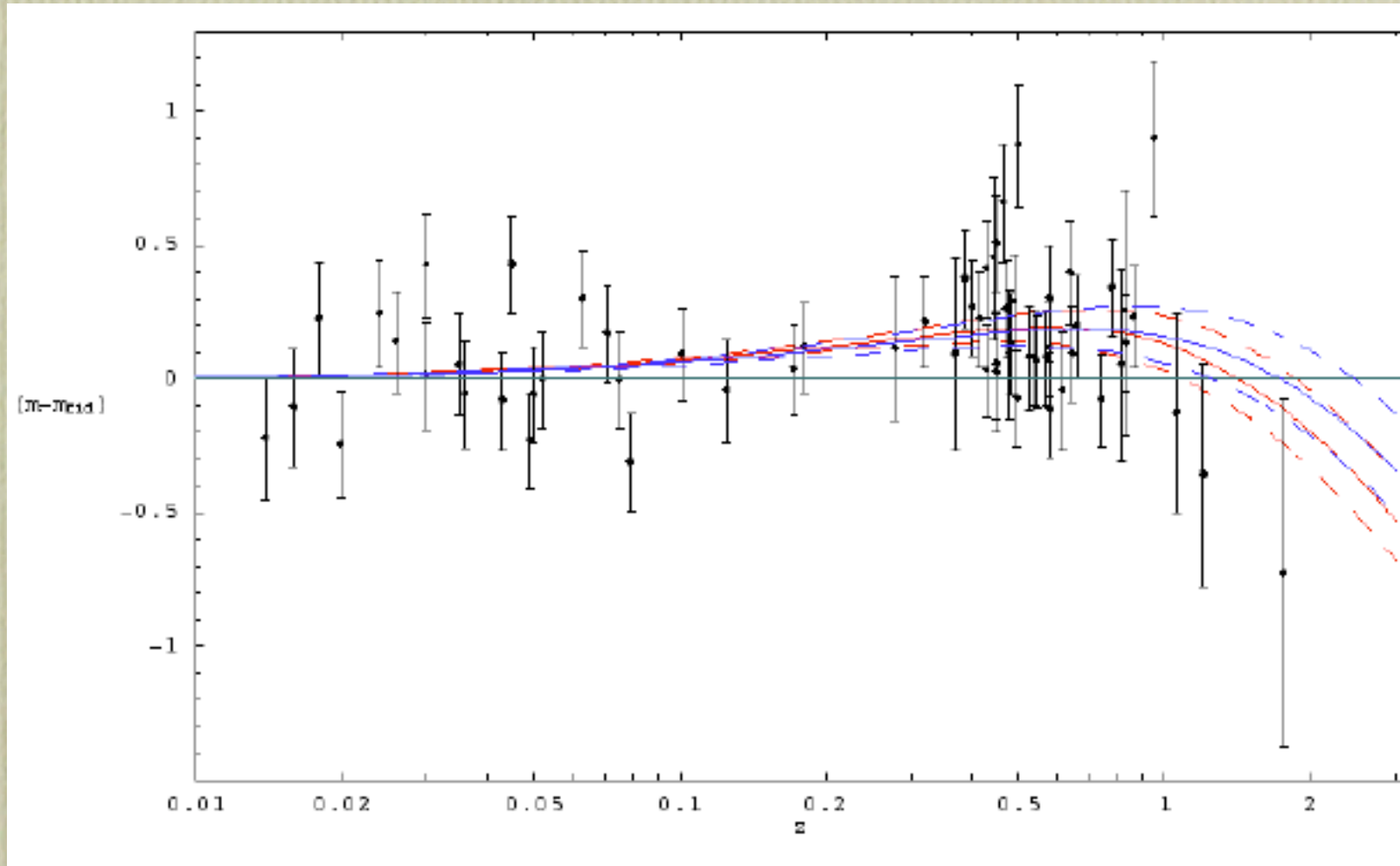


Compare with Observation

CMB Power Spectrum: parameter is m



Type Ia Supernovae: no free parameters



Conclusions

- Predictions so far agree with observation
- Makes definite predictions also with regard to galaxy number counts
- Results are different from predictions coming from a nonzero cosmological constant (e.g., see the low multipole power spectrum of the CMB).
- Observations will be good enough within 5 years to rule out one or both of these models.