Gauge-Invariant Approach to Homogeneous Cosmological Perturbations

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In this talk we use the gauge-invariant approach to study the question on the impact of homogeneous cosmological fluctuations to the expansion rate of the universe.

Summary

1. Homogeneous perturbations - a kinematical approach
2. Gauge-invariant approach
3. Instability in the expansion rate
4. Conclusion

1. Homogeneous perturbations

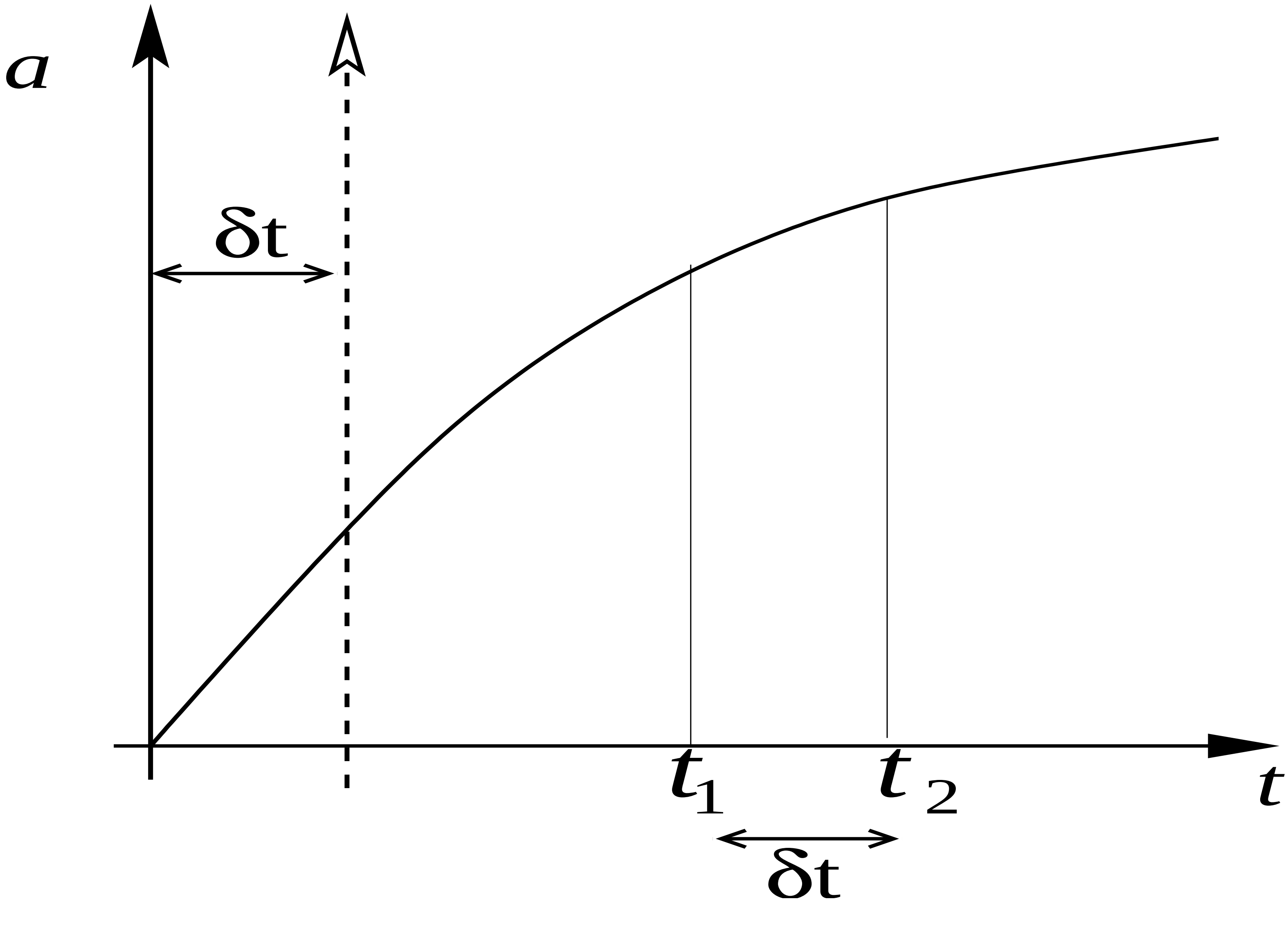
$$\delta\rho = \delta\rho(t)$$

$$\rho(t, \vec{x}) = \rho_o(t_o, \vec{x}) + \delta\rho(t)$$

$$\delta\rho(t) = \dot{\rho}(t_o)\,\delta t$$

Can it be a mere shift in the time coordinate?

1.1 Kinematical Approach



$$
\begin{aligned}
a(t_1) &= a(t_2) - \dot{a}(t_2)\,\delta t \qquad \delta t > 0 \\
\dot{a}(t_1) &= \dot{a}(t_2) - \ddot{a}(t_2)\,\delta t \\
\ddot{a}(t_1) &= \ddot{a}(t_2) - \dddot{a}(t_2)\,\delta t
\end{aligned}
$$

$$
H_1 \equiv H(t_1) \equiv \frac{\dot{a}(t_1)}{a(t_1)} \qquad - q_1 \equiv \frac{\ddot{a}(t_1)a(t_1)}{[\dot{a}(t_1)]^2}
$$

$$H_1 = H_2 + \underbrace{\left((H_2)^2 - \frac{\ddot{a}_2}{a_2} \right)}_{\delta H > 0} \delta t$$

$$-q_1 = -q_2 + \underbrace{\left(\frac{\dddot{a}_2 \, a_2}{(\dot{a}_2)^2} + \frac{\ddot{a}_2}{\dot{a}_2} - 2\frac{(\ddot{a}_2)^2 a_2}{(\dot{a}_2)^3} \right)}_{\delta q} \delta t$$

Sure, when

$$a \sim t^n$$

then

$$\delta q = 0$$

(as expected, since that implies $q = const.$)

But it is a different constant for each power law:

$$a = a_o t^n \qquad \Longrightarrow \qquad -q = \frac{n-1}{n}$$

It thus seems natural to blame a change of the decceleration parameter on the change of the equation of state of the background fluid.

Note that it may be a composite fluid, and such a change may well indicate the change of dominance of one component over the other.

Nevertheless, the previous approach lacks formality. It cannot show exactly how a change in the equation of state affects q.

We will recall now some basic perturbation equations, expand the perturbations in harmonics as usual and then take the long-wavelength limit.

Previous works [Kolb et al] having perturbed the metric itself, obtain gauge-dependent results. Although the results must coincide once the gauge is fixed, it is a good policy to do it explicity and as late as possible in the calculations.

2. Gauge-invariant Approach

Raychaudhuri:

$$\dot{\theta} + \frac{\theta^2}{3} + 2\sigma^2 + 2\omega^2 - a^{\alpha}{}_{;\alpha} = -\frac{1}{2}\rho(1 + 3\lambda)$$

Conservation, projected:

$$\dot{\rho} + (\rho + p)\theta + \dot{q}^{\mu}V_{\mu} + q^{\alpha}{}_{;\alpha} - \pi^{\mu\nu}\Theta_{\mu\nu} = 0$$

Perturbed Equations[a]

$$(\delta\theta)^{\bullet} + \dot{\theta}\delta V^0 + \frac{2}{3}\theta\,\delta\theta - (\delta a^{\alpha})_{;\alpha} = -\frac{1}{2}(1+3\lambda)\delta\rho$$

$$(\delta\rho)^{\bullet} + \dot{\rho}\,\delta V^0 + \theta(1+\lambda)\delta\rho + (1+\lambda)\rho\,\delta\theta + (\delta q^{\alpha})_{;\alpha} = 0$$

where we have assumed $\delta p = \lambda\,\delta\rho$.

[a] *Minimal Closed set of observables in the theory of cosmological perturbations*, M. Novello, J.M. Salim, M.C. Motta da Silva, **S.E.J.**., R. Klippert, Phys. Rev. **D51**, 2 (1995) 450-461, gr-qc/9403014

Expansion in Harmonics
— Scalar Perturbations

$$\nabla^2 Q^{(k)}(\vec{x}) = -k^2\, Q^{(k)}(\vec{x})$$

$$Q^{(k)}_{ij} \equiv Q^{(k)}_{,i;j}$$

$$Q^{(k)}_{i} \equiv Q^{(k)}_{,i}$$

$$\hat{Q}^{(k)}_{ij} \equiv Q^{(k)}_{ij} + \frac{k^2}{3} Q^{(k)}\ \gamma_{ij}$$

$$
\begin{aligned}
\delta a_i &\equiv \sum_k \Psi^{(k)}(t)\, Q_i^{(k)} \quad , \\
\delta q_i &\equiv \sum_k q^{(k)}(t)\, Q_i^{(k)} \qquad \text{HEAT FLUX!} \qquad (1) \\
\delta V_j &\equiv \sum_k V^{(k)}(t)\, Q_j^{(k)} \quad , \\
\delta V_0 &\equiv \sum_k \beta^{(k)}(t)\, Q^{(k)} \quad , \\
\delta \rho &\equiv \sum_k N^{(k)}(t)\, Q^{(k)} \quad , \\
\delta \theta &\equiv \sum_k H^{(k)}(t)\, Q^{(k)} \quad .
\end{aligned}
$$

(2)

$$\dot{H} - \frac{1}{2}\dot{\theta}\beta + \frac{2}{3}\theta H + k^2\Psi = -\frac{1}{2}(1+3\lambda)N$$

$$\dot{N} - \frac{1}{2}\dot{\rho}\beta + \theta(1+\lambda)N + \rho(1+\lambda)H - k^2 q = 0$$

upon which we make

$$k \rightarrow 0 \qquad \text{and}$$

$$\beta = 0 \qquad \text{GAUGE FIXING!} \quad (\delta V_0 = 0)$$

which is reasonable for a dust-dominated universe: the coefficient of β behaves like t^{-3}, much faster than other terms in the same equation.

We get then

$$\ddot{H} - \underbrace{\left[\frac{2}{3}\theta^2 + \frac{1}{6}(1+3\lambda)(5+3\lambda)\rho\right]}_{\equiv A(t)} H = [\,\underbrace{\frac{1}{6}(1+3\lambda)(5+3\lambda)\theta}_{\equiv B(t)} - \frac{3}{2}\dot{\lambda}]\; N$$

$$\begin{aligned} A(t) &\sim t^{-2} \\ B(t) &\sim t^{-1} \end{aligned}$$

3. Instability

$$A(t) > 0 \quad \forall t :$$

INSTABILITY!

Also, if

$$\begin{aligned} \dot{\lambda} &< 0 \\ N &> 0 \qquad \text{(overdensity)} \end{aligned}$$

then we get a positive source!

$$\ddot{H} - A(t)H = [B(t) - \frac{3}{2}\dot{\lambda}]\, N$$

4. Conclusion

- A sudden change in the equation of state may cause a large change in the expansion rate
- We are looking into a more realistic model, with two fluids