Lecture 4

The Beta Function and Renormalization Group Flow

In the previous lecture we have seen how to compute the beta function by using the renormalization procedure resulting in the counterterms of the theory. The Callan-Symanzik equation is particularly useful in the context of perturbation theory. However, the content of the beta function goes well beyond this. It gives us the flow of a quantum field theory from one scale to another, independently of whether this is perturbative or not. Let us consider the possible behaviors as classified by the sign of $\beta(g)$, with g the corresponding coupling.

$\underline{\beta(g) > 0}$:
This is the case of theories like the ϕ^4 in four dimensions as well as QED at weak coupling.

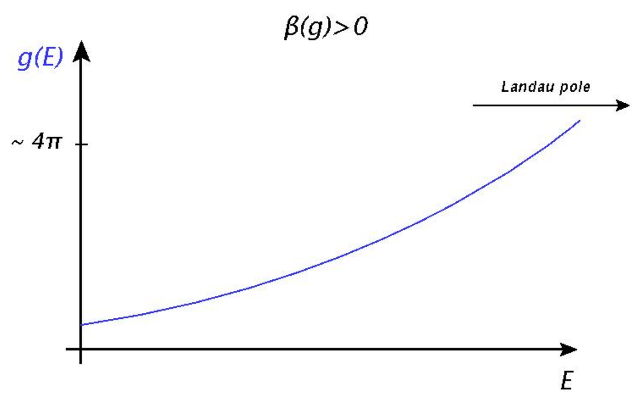

Figure 4.1: Evolution of a coupling with energy scale in theories with $\beta(g) > 0$. This is the case of QED and also ϕ^4. As the coupling grows it becomes non-perturbative (here denoted by $\sim 4\pi$ for the case for four-dimensions). In the case of the QED coupling, or the coupling of any $U(1)$ gauge theory, there is a Landau pole in the ultra-violet.

In Figure 4.1 we schematically show the behavior of a coupling with positive $\beta(g)$. As long as the coupling remains perturbative (e.g. below $\sim 4\pi$ for a dimensionless coupling as λ or the QED coupling in four-dimensions) we can trace the growth of the coupling using perturbation theory computations. For the case of the QED coupling e, or the coupling of any $U(1)$ gauge theory, there is Landau pole in the ultra-violet.

$\underline{\beta(g) = 0}$:
In this case the coupling g is independent of the scale, i.e $g =$ constant. Scale invariant (or more generally conformally invariant) theories are useful as limits of the renormalization group flow. But they do not have a particle spectrum. Nonetheless we can know some aspects of the behavior of the correlation functions directly by imposing scale invariance on them. One of the most studied theories in this category is $\mathcal{N} = 4$ Super Yang-Mills, where the $\mathcal{N}$ refers to the number of supersymmetries of the theory. This conformal field theory satisfies $\beta(g) = 0$ for its supersymmetric gauge theory and is at the heart of the AdS/CFT correspondence, being the boundary theory.

$\underline{\beta(g) < 0}$:
In this case the coupling decreases for increasing energies. In fact it goes to zero at infinitely high energy. This is what is called asymptotic freedom. As an example, let us consider the beta function to first order in the coupling g as given by

$$\beta(g) = -\frac{1}{2}\, C\, g^3 \;, \tag{4.1}$$

where the constant is $C > 0$ and we neglect higher orders starting at g^5. Then, we have

$$\frac{dg}{d\ln\mu} = -\frac{1}{2}\, C\, g^3 \;, \tag{4.2}$$

resulting in

$$\int_{\mu_R}^{q} \frac{dg}{g^3} = -\frac{C}{2}\,\ln\left(\frac{q}{\mu_R}\right) \;, \tag{4.3}$$

where μ_R is some reference scale. Then, the coupling is

$$g^2(q) = \frac{g^2(\mu_R)}{1 + C\, g^2(\mu_R)\, \ln(q/\mu_R)} \;. \tag{4.4}$$

We see that the coupling decreases logarithmically towards the UV and it would actually reach a UV fixed point for $q \to \infty$. On the other hand, as we see in Figure 4.2, the coupling

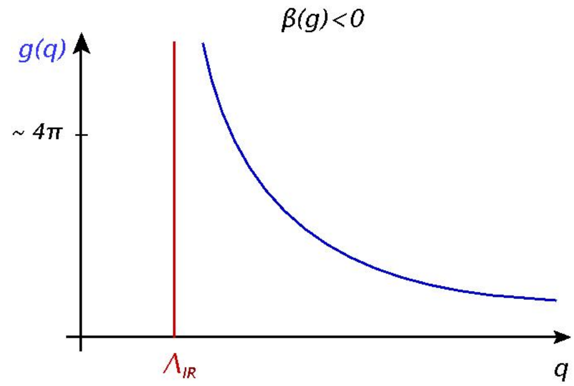

Figure 4.2: Evolution of a coupling with energy scale in theories with $\beta(g) < 0$. This is the case of many non-abelian gauge theories. In particular it is the case of QCD.

growths towards the IR up to a point in which becomes non-perturbative. The infrared scale $\Lambda_{\rm IR}$ signals not just the breakdown of perturbation theory but also the scale at which the perturbative degrees of freedom might be confined. For instance, in QCD quark and gluons become strongly coupled and confine below an infrared hadronization scale signaling the appearence of hadrons instead. In principle, there can be other phenomena associated with this infrared strong coupling, such as chiral symmetry breaking in the low energy limit of the strong interactions.

4.1 Fixed Points

The vanishing of the beta function results in a fixed point of the renormalization group evolution. At the fixed point scale transformations leave the theory invariant since the coupling does not vary under them. In Lecture 2 we introduced the Gaussian or free theory fixed point. In the previous section we saw that an asymptotically free theory would reach a UV fixed point at $q \to \infty$ (see Figure 4.2).

It is also possible to have more interesting, non-trivial fixed points. For instance, let us consider a theory with a beta function that is positive for very small values of the coupling g. This is for instance the case of ϕ^4 theory. It is possible to imagine that, as the coupling growths larger (i.e. at strong coupling), the beta function would change sign. This would correspond to the higher order terms overpowering the leading order one, although we can consider the more general case where as g grows we loose perturbativity.

The situation is schematically illustrated in Figure 4.3. The arrows indicate the behavior of the coupling $g(\mu)$ as μ grows towards the UV. As long as $\beta(g) > 0$, the coupling $g(\mu)$ grows with μ. But as $\beta(g)$ crosses zero and changes signs, then this is revered and $g(\mu)$ decreases its value as μ moves towards the UV. Thus, we see that $g(\mu) = g_*$ is a UV-stable fixed point of this theory. The renormalization group flows toward this fixed point g_* in

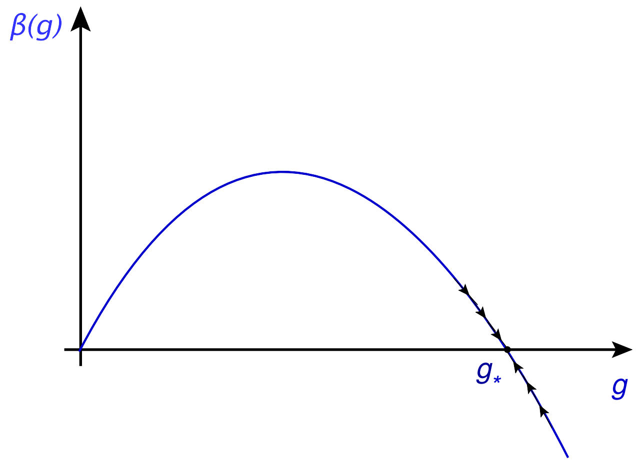

Figure 4.3: An example of a UV-stable fixed point. While $\beta(g) > 0$ the coupling $g(\mu)$ grows with the energy scale μ. But once $\beta(g)$ crosses zero and $\beta(g) < 0$, the flow (indicated by the arrows) reverses and $g(\mu)$ decreases towards the UV.

the UV. Close to the fixed point we can linearize the behavior of the beta function by approximating it as

$$\beta(g) \simeq -B(g - g_*), \tag{4.5}$$

with $B > 0$. As before, we integrate between a reference scale μ_R and q.

We then obtain

$$\int_{\mu_R}^{q} \frac{dg}{(g - g_*)} = -B \ln\left(\frac{q}{\mu_R}\right) = \ln\left(\frac{q}{\mu_R}\right)^{B} , \tag{4.6}$$

which results in

$$g(q) \simeq g_* + (g(\mu_R) - g_*) \left(\frac{\mu_R}{q}\right)^{B} . \tag{4.7}$$

The expression (4.7) clearly displays the expected behavior, i.e.$g(q) \to g_*$ as $q \to \infty$.

Similarly, it is possible to have an IR-stable fixed point. This corresponds to the situation depicted in Figure 4.4. We see that while $\beta(g) < 0$ the coupling grows as the energy scale μ *decreases*. This flow towards the IR is indicated by the arrows. On the other hand, when $\beta(g) > 0$ the opposite is true: the coupling decreases as μ moves towards the IR. Thus, in this example g_* is an IR-stable fixed point.

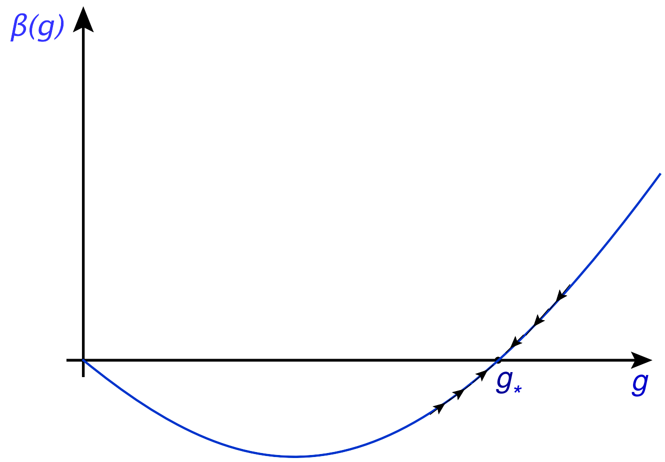

Figure 4.4: An example of a IR-stable fixed point. While $\beta(g) < 0$ the coupling $g(\mu)$ growths as the energy scale μ decreases toward the IR. Once $\beta(g)$ crosses zero and $\beta(g) > 0$, the flow (indicated by the arrows) reverses and $g(\mu)$ decreases towards the IR.

4.2 An Example: Ferromagnetism through the Renormalization Group

Here we consider the large scale (IR) description of ferromagnet using a scalar theory to model the magnetization. In general, when we describe the long range behavior of a physical system we will be interested in an array of physical properties. We will see that this behavior close to a fixed point is power-like. For instance, the correlation length ξ in a ferromagnetic material behaves like

$$\xi \sim \left(\frac{T-T_c}{T_c}\right)^{-\nu} , \tag{4.8}$$

where T_c is the critical temperature, and ν is called a critical exponent. Similar critical exponents can be defined for the heat capacity, the magnetization and other physical quantities. Then the two-point correlation function behaves like

$$G(x,y) \sim \begin{cases} e^{-|\mathbf{x}-\mathbf{y}|/\xi} , & \text{for } |\mathbf{x}-\mathbf{y}| \gg \xi \\ \frac{1}{|\mathbf{x}-\mathbf{y}|^{d-2+\eta}} , & \text{for } |\mathbf{x}-\mathbf{y}| \ll \xi \ . \end{cases} \tag{4.9}$$

where the exponent η is the anomalous dimension of the field ϕ. As is the case with many other physical systems with a scalar order parameter, a ferromagnet can be described by the action

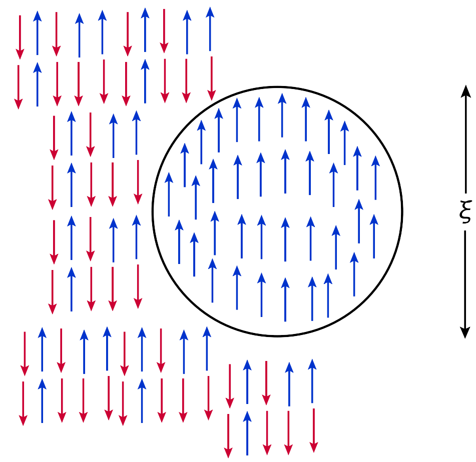

Figure 4.5: The correlation length ξ in a ferromagnet.

$$S[\phi] = \int d^d x \left\{ \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!}\phi^4 \right\} . \tag{4.10}$$

This is the Landau-Ginzburg model of a ferromagnet. Higher dimensional operators can be neglected since we are interested only in relevant operators. We will first consider the mean field theory approximation leading to the conditions for the ferromagnetic transition. Then will include the effects of fluctuations through the renormalization group evolution.

Mean Field Theory:

We make the simple assumption of a spatially uniform field configuration, i.e.

$$\partial_\mu \phi = 0 , \tag{4.11}$$

and then minimize the energy of the system. This results in

$$m^2 \bar{\phi} + \frac{\lambda}{6}\bar{\phi}^3 = 0 , \tag{4.12}$$

where $\bar{\phi}$ is the value of the (spatially constant) field at the minimum. The two solutions to (4.12) are

$$\bar{\phi} = \begin{cases} 0 \\ \sqrt{\frac{-6\, m^2}{\lambda}} \end{cases} . \tag{4.13}$$

The first solution corresponds to the paramagnetic phase. The second one is only accessible if $m^2 < 0$. For the ferromagnetic system (as well as many other systems described by this or similar scalar actions), m^2 is temperature dependent

$$m^2 = c\,(T - T_c)\ , \tag{4.14}$$

where c is a constant, and again T_c is the critical temperature, this time corresponding to the ferromagnetic transition. So for $T < T_C$ we can have

$$\bar{\phi} \neq 0\ , \tag{4.15}$$

corresponding to a nonzero value of the macroscopic magnetization. As we will see later in the lectures, this is a first example of a mechanism called spontaneous symmetry breaking. However, for now we are only interested in understanding if the simple analysis from the mean field approximation is robust against the inclusion of fluctuations.

<u>Renormalization Group Evolution</u>:

Now we consider the effect of integrating out the high energy modes and the rescaling of lengths, i.e. the renormalization group (RG) evolution of the LG theory of a ferromagnet. This will lead to RG equations for m^2 and λ which we can use to better understand the phases of the system. We start by splitting the low and high energy modes as in

$$\phi(x) = \phi_\ell(x) + \phi_h(x)\ , \tag{4.16}$$

where the low energy fields $\phi_\ell(x)$ correspond to momenta satisfying $k < \Lambda/b$, the high energy fields $\phi_h(x)$ have Fourier components with momenta $\Lambda/b < k < \Lambda$. Here Λ is the momentum cutoff and we take $b > 1$. Just as we did earlier, we integrate out the high momentum modes $\phi_h(x)$ to obtain the low energy effective theory as in

$$e^{-S_{\rm eff.}[\phi_\ell]} = e^{-S[\phi_\ell]} \int \mathcal{D}\phi_h\, e^{-S[\phi_\ell,\phi_h]}\, e^{-S[\phi_h]}\ , \tag{4.17}$$

where $S[\phi_\ell, \phi_h]$ is the part of the action (4.10) depending on both ϕ_ℓ and ϕ_h and it comes exclusively from the interaction term, i.e. it vanishes with λ. The last factor in (4.17) will become a mere normalization once the integral over ϕ_h is performed. Integrating out ϕ_h at leading order in perturbation theory corresponds to performing the loop calculations in Figure 4.6.

For instance, the contribution of diagram (a) in Figure 4.6 results in a shift in the two-point function given by

Figure 4.6: Integrating out the high energy modes. The double lines correspond to ϕ_h, whereas the external single lines are ϕ_ℓ, the low energy modes. (a): first order contribution to the ϕ_ℓ two-point function. (b): lowest order contribution to the four-point function. These diagrams result in shifts in λ and m^2.

$$\frac{\lambda}{4} \int_{\Lambda/b}^{\Lambda} \frac{d^d k}{(2\pi)^d} \frac{1}{m^2 + k^2} \int^{\Lambda/b} \frac{d^d q}{(2\pi)^d} \phi_\ell(q)\, \phi_\ell(-q) \ . \tag{4.18}$$

Unlike what we did earlier, where we had consider the case $m^2 = 0$, we now allow for a nonzero mass. However, since m^2 is a mass term in the IR too, it is only a small perturbation inside the integral over the high energy interval. So both m^2 and λ are perturbations of the Gaussian fixed point, albeit relevant and marginal ones. Then the shift in m^2 is

$$\delta m^2 = \frac{\lambda}{2} \left(I_1 - m^2 I_2\right) \ , \tag{4.19}$$

where we defined

$$I_i = \int_{\Lambda/b}^{\Lambda} \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2)^i} \ . \tag{4.20}$$

But to obtain the full contribution of the renormalization flow to m^2 we also need to shift the scales. That is, we shift

$$x \to x/b = x' \ , \tag{4.21}$$

and also the associated shift in momentum $k \to bk$. This will results in rescalings in all the parameters of the theory. The field rescaling is

$$\phi \to b^{d_\phi}\, \phi \ , \tag{4.22}$$

where d_ϕ is the dimension of the field ϕ. This is chosen so as to leave the kinetic term invariant, the actual definition of the Gaussian fixed point. Let us do this explicitly. We need to impose the invariance on

$$\int d^d x \frac{1}{2} \partial_\mu \phi \partial_\mu \phi \ . \tag{4.23}$$

the volume factor rescales as $d^d x \to b^d d^d x'$. The derivative is

$$\partial_\mu = \frac{\partial}{\partial x_\mu} \to b^{-1} \frac{\partial}{\partial x'_\mu} = b^{-1}\, \partial'_\mu \ . \tag{4.24}$$

Thus, the invariance of (4.23) requires that

$$d - 2 - 2d_\phi = 0 \ , \tag{4.25}$$

which results in

$$\boxed{d_\phi = \frac{d-2}{2}} \ . \tag{4.26}$$

This implies that just under the simple rescaling of distance scales the mass squared behaves as

$$m^2 \to b^{d-2d_\phi} m^2 = b^2 m^2 \ . \tag{4.27}$$

Then, the shift in m^2 from both the rescaling of length scales and the integrating out of high energy modes is given by

$$m^2 \to b^2 \left[m^2 + \delta m^2 \right] \ , \tag{4.28}$$

with δm^2 given by (4.19).

We follow a similar procedure to obtain the shift in the coupling λ from integrating out the high momentum modes in the diagram (b) of Figure 4.6 and rescaling the length scales. The ϕ^4 term in the action now scales when $x \to x/b = x'$ so that we have

$$\lambda \to b^{4-d}\, \lambda \ . \tag{4.29}$$

The loop integral by itself will result in a shift given by

$$\frac{\delta\lambda}{4!} = -\frac{\lambda^2}{16}\int_{\Lambda/b}^{\Lambda} \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2+m^2)^2} , \tag{4.30}$$

To understand the sign we need to remember that this term comes from the perturbative expansion of the term $S[\phi_\ell, \phi_h]$ in (4.17). This is

$$\begin{aligned} e^{-S[\phi_\ell,\phi_h]} &= e^{-(\lambda/4)\int d^dx\phi_\ell^2\,\phi_h^2} \\ &= 1 - \frac{\lambda}{4}\int d^dx\phi_\ell^2(x)\phi_h^2(x) + \frac{1}{2!}\frac{\lambda^2}{16}\int d^dx\phi_\ell^2(x)\phi_h^2(x)\int d^dy\phi_\ell^2(y)\phi_h^2(y) + \ldots \end{aligned} \tag{4.31}$$

The third term in (4.31) corresponds to a negative contribution to $S_{\text{eff.}}$, as it appears in (4.30). Combining (4.30) and (4.29) we see that the shift in the coupling λ from the renormalization flow is

$$\lambda \to b^{d-4}\left[\lambda + \delta\lambda\right] . \tag{4.32}$$

Performing the momentum integrals in (4.19) and (4.30) we obtain δm^2 and $\delta\lambda$. But as these stand here, they will depend on the cutoff Λ. To eliminate this dependence, which only obscures the calculation, we will rescale the momenta to momenta in units of the cutoff Λ. This means we will make the replacement

$$k \to \frac{k}{\Lambda} , \tag{4.33}$$

in all momenta. This also means to measure length scales in units of the inverse cutoff $a = \Lambda^{-1}$ (the "lattice" spacing). This rescaling is $x \to x\,\Lambda$. Since the new momentum and length scales are dimensionless (measured in units of the cutoff Λ) all dimensionfull parameters in the action must be replaced by dimensionless ones measured in terms of Λ to the power dictated by the dimensions of this quantity. In particular we must make the replacements

$$\begin{aligned} m^2 &\to m^2\,\Lambda^2 \\ \lambda &\to \lambda\,\Lambda^{4-d} , \end{aligned} \tag{4.34}$$

where m^2 and λ on the right are now dimensionless. With this replacement, we can now obtain the renormalization group equations for m^2 and λ using (4.28) and (4.32). For this purpose we expand around four dimensions by defining

$$\epsilon \equiv 4 - d \ , \tag{4.35}$$

and expanding around $\epsilon \sim 0$. Keeping only leading order terms in the ϵ expansion and taking derivatives of the shifts for m^2 and of λ with respect to the $\ln b$ we obtain

$$\frac{dm^2}{d\ln b} = 2m^2 + \frac{\lambda}{16\pi^2} - \frac{m^2\lambda}{16\pi^2} \tag{4.36}$$

$$\frac{d\lambda}{d\ln b} = \epsilon\lambda - \frac{3}{16\pi^2}\lambda^2 \ . \tag{4.37}$$

These are the beta functions for m^2 and λ, $\beta(m^2)$ and $\beta(\lambda)$. There are defined as logarithmic derivatives with respect to the rescaling of scales. As b growths, the lower bound on the high momentum integral is lowered, indicating a flow to lower momenta and larger length scales (also we replace x with bx'). So we expect these beta functions to have the opposite signs compared with the ones we computed earlier with respect to the logarithmic derivative of energy scales. For instance, the second term in (4.37) corresponds to the beta function of the ϕ^4 theory we computed before for the d=4 case (i.e. $\epsilon = 0$), only that was defined in terms of the variation of energy/momentum scales, so the sign is inverted.

More importantly, we see that in general, for $\epsilon > 0$ we have interesting non-trivial fixed points. First the Gaussian fixed point corresponds to $(m^2 = 0, \lambda = 0)$, for which $\beta(m^2) = 0$ and $\beta(\lambda) = 0$. On the other hand, for $d < 4$ we see that there is a fixed point for $\beta(\lambda)$ for

$$\lambda_* = \frac{16\pi^2}{3}\epsilon \ . \tag{4.38}$$

Using the type of analysis from previous sections we see that this corresponds to an IR-stable fixed point. That is, the system flows to it at longer length scales. Furthermore, at the λ fixed point there is also a fixed point for m^2. Replacing (4.38) in (4.36) we obtain

$$m_*^2 = -\frac{\epsilon}{6} \ , \tag{4.39}$$

where we neglected the term of order $m^2\epsilon$ as a higher order in the perturbation expansion. Thus, $\beta(m^2)$ is *negative* for $m^2 < m_*^2$ and it switches to *positive* as we cross to $m^2 > m_*^2$. This means that this fixed point is repulsive as we move to the IR, i.e. the system flows away from m_*^2 in the IR limit. The interplay between these two fixed points, the one for λ and the one for m^2 has important consequences for the behavior of the system in the IR. This can be seen in the phase diagram of Figure 4.7.

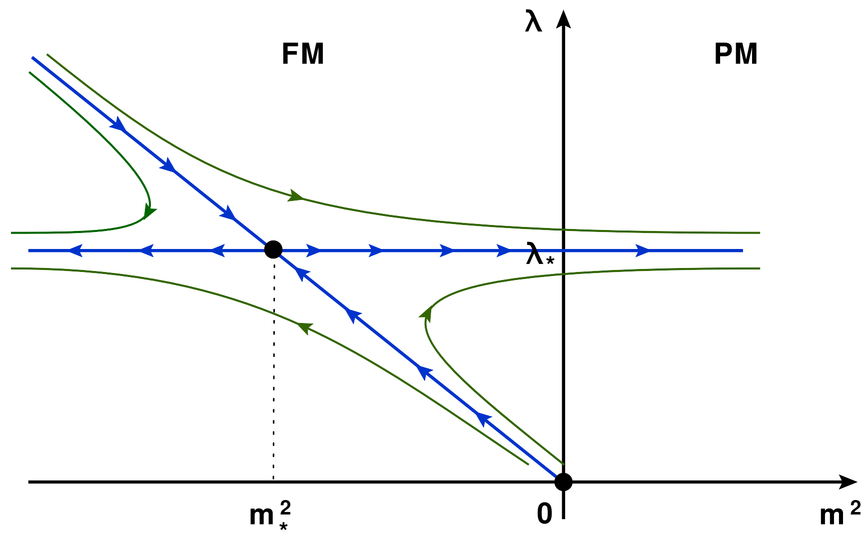

Figure 4.7: Phase diagram of the ferromagnetic transition. The blue lines are the critical surface. The green lines are RG flows for various values of m^2 and λ. FM and PM stand for ferromagnetic and paramagnetic phases respectively.

The blue lines represent the critical flow. For instance, starting at the Gaussian fixed point, a small negative value of m^2 will drive the system towards the non-trivial IR stable fixed point. The same is true for a large negative value of m^2 as long as we are on the blue line. This is just the projection onto the $\beta(\lambda)$ vs. λ plot, i.e. the behavior of $\beta(\lambda)$. Similarly, the horizontal blue line denotes the behavior of $\beta(m^2)$, i.e. is repulsive with respect to the non-trivial fixed point. The green lines denote the flow of the system around these critical lines. We see that, for a given value of m^2, even if it is initially negative, there is always a value of λ large enough that the system will flow towards the paramagnetic phase in the IR. So we see that considering the effects of the interactions, the simple mean field theory result that there is a ferromagnetic phase as long as $m^2 < 0$ is modified. The renormalization group analysis provides us with a picture of the phase diagram that includes the effects of the fluctuations.

4.3 The Correlation Length Critical Exponent

We want to consider the behavior of the correlation length ξ close to a fixed point. The distance to the fixed point is controlled by the coupling constant, which we generically call λ, corresponding to a local operator. By dimensional analysis we can write

$$\xi = af(\lambda) \ , \tag{4.40}$$

where a is a typical length scale of the system such as the inverse cutoff (lattice spacing) and $f(\lambda)$ is a dimensionless function of the coupling λ. To obtain a Callan-Symanzik equation for the correlation length we use the fact that physical dimensionfull quantities must be invariant under the action of the renormalization group flow, This means

$$\frac{d\xi}{\ln a} = 0 \ . \tag{4.41}$$

This results in

$$a\, f(\lambda) + a\, \frac{\partial f}{\partial \lambda}\, \frac{\partial \lambda}{\partial \ln a} = 0 \ . \tag{4.42}$$

Then, we obtain an RG equation for $f(\lambda)$ given by

$$f(\lambda) + \frac{\partial f}{\partial \lambda}\, \beta(\lambda) = 0 \ . \tag{4.43}$$

It is straightforward to integrate (4.43) to obtain

$$f(\lambda) = f(\lambda_0)\, e^{\int_{\lambda}^{\lambda_0} \frac{d\lambda'}{\beta(\lambda')}} \ , \tag{4.44}$$

with λ_0 some arbitrary value of the coupling. We are interested in the correlation length ξ for any two values of the coupling. Then we have

$$\frac{\xi(\lambda_1)}{\xi(\lambda_2)} = \frac{f(\lambda_1)}{f(\lambda_2)} = e^{\left[\int_{\lambda_1}^{\lambda_0} \frac{d\lambda'}{\beta(\lambda')} - \int_{\lambda_2}^{\lambda_0} \frac{d\lambda'}{\beta(\lambda')}\right]} = e^{\int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\beta(\lambda)}} \ , \tag{4.45}$$

so, in sum we have

$$\boxed{\xi(\lambda_1) = \xi(\lambda_2)\, e^{\int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\beta(\lambda)}}} \ . \tag{4.46}$$

Now, close to the fixed point we can write the beta function as

$$\beta(\lambda) = \beta'(\lambda_*)\, (\lambda - \lambda_*) + \mathcal{O}((\lambda - \lambda_*)^2) \ , \tag{4.47}$$

where in the Taylor expansion around λ_* we already used the fact that $\beta(\lambda_*) = 0$ and $\beta'(\lambda_*)$ is the first derivative of the beta function evaluated at the fixed point coupling. Let us consider the case with $|\lambda_1 - \lambda_*| \ll \lambda_*$, i.e. for λ_1 very close to the fixed point. On the other hand, we take λ_2 at a finite distance from λ_*, but still close enough so that the approximation (4.47) is valid. Then we can see that

$$\xi(\lambda_1) = \xi(\lambda_2)\, e^{\int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\beta'(\lambda_*)(\lambda-\lambda_*)+\ldots}} = \xi(\lambda_2)\, e^{\frac{1}{\beta'(\lambda_*)} \ln\left(\frac{\lambda_2-\lambda_*}{\lambda_1-\lambda_*}\right)} \tag{4.48}$$

which results in

$$\xi(\lambda_1) = \xi(\lambda_2) \left(\frac{\lambda_2 - \lambda_*}{\lambda_1 - \lambda_*}\right)^{1/\beta'(\lambda_*)} . \tag{4.49}$$

The expression above tells us that as $\lambda_1 \to \lambda_*$ the correlation length diverges with a power determined by the inverse of the first derivative of the beta function evaluated at the fixed point value. This is a first example of a critical exponent. It is customary to define the critical exponent ν for the correlation length such that for our example it would be

$$\xi(\lambda) \sim (\lambda - \lambda_*)^{\nu} . \tag{4.50}$$

Then the critical exponent for the correlation length is

$$\boxed{\nu = -\frac{1}{\beta'(\lambda_*)}} . \tag{4.51}$$

In the particular case of the correlation length, its divergence close to the fixed point typically signals a phase transition. The renormalization group approach gives us an expression for the critical behavior. As we will see later, this behavior is universal in the sense that it does not depend crucially on microscopic details of the system.

Additional suggested readings

- *An Introduction to Quantum Field Theory*, M. Peskin and D. Schroeder, Section 12.3.
- *Condensed Matter Field Theory*, Altland and Simons, Section 8.4.