Quantum Field Theory II

Homework 3
Due 15/01/2024

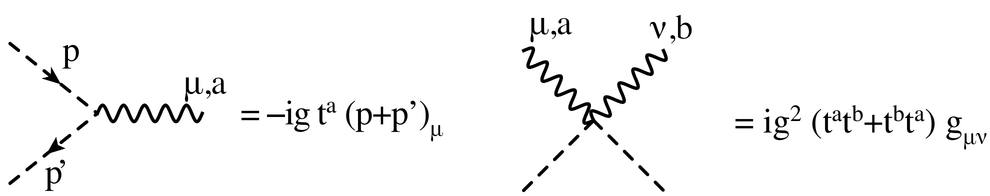
1. Consider a non-abelian gauge theory coupled to a fermion field (**4 pts.**):

   (a) Compute at one loop level the counter-term that regularizes the fermion field (δ_2).

   (b) Also at one loop, compute the counter-term regularizing the vertex. (δ_1).

   (c) Using the result obtained in class for δ_3 compute the one-loop beta function $\beta(g)$.

2. **Non-Abelian Scalar Field (3pts.)**:
   Consider a non-abelian gauge theory with a gauge group G. Couple to it a complex scalar field in the representation r (instead of a fermion). The Feynman rules of the theory refering to the interaction are:



where the generators t^a correspond to the representation r. Compute the β function of the theory. Show that it is given by

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left(\frac{11}{3}C_2(G) - \frac{1}{3}C(r)\right)$$

where $\mathrm{Tr}\left[t^a t^b\right] = C(r)\delta^{ab}$ and $C_2(G)$ is the quadratic Casimir operator of the adjoint representation, i.e. $f^{acd}f^{bcd} = C_2(G)\delta^{ab}$.

Hint: Use that $(\delta_1 - \delta_2)$ in a non-abelian gauge theory is universal, i.e. it does not depend on the matter content.

3. **Not the Standard Model (3pts.)** : Consider an $SU(2)$ gauge theory with lagrangian

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi - V(\phi^\dagger \phi) \; - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu},$$

where the scalar field ϕ transforms in the *adjoint representation* of $SU(2)$. Thus, the covariant derivative is given by

$$(D_\mu \phi)^a = \left(\partial_\mu \phi^a + g \, \epsilon^{abc} \, A^b_\mu \, \phi^c \right) \, ,$$

with g the $SU(2)$ coupling constant, $a, b, c = 1, 2, 3$ are the group indices and ϵ^{abc} is the Levi-Civitta tensor in three dimensions, i.e. the $SU(2)$ structure constants. Choose a vacuum with only one of the three components of $\phi^a(x)$ non zero, for instance

$$\langle \phi^3 \rangle = v, \qquad \langle \phi^1 \rangle = \langle \phi^2 \rangle = 0 \; .$$

Compute the resulting gauge boson masses. What is the symmetry breaking pattern ? I.e. Is there an remnant *unbroken* gauge symmetry ? What is it ?

Note: This is the Georgi-Glashow model. For a while it was a possible candidate for electroweak unification. Why was it discarded ?