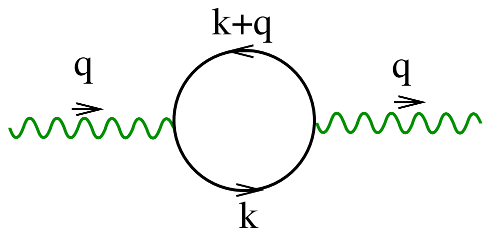
Quantum Field Theory II

Homework 1
Due 22/09/2023

1. **Renormalizatio of the QED Interaction**: (3.5 pts).
   We saw in class that the renormalization of the QED coupling is fully determined by the computation of the photon 2-point function counterterm, δ_3. Compute this counterterm at one loop order, by computing the diagram in the figure and imposing a suitable renormalization condition. Explain the procedure in detail.



2. **Beta Functions of Yukawa Theory**: (3 pts.)
   Consider the point 4. of Homework 5 of the first semester, the Yukawa lagrangian with pseudoscalar coupling:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \bar{\psi}(i\,\partial\!\!\!/ - M)\psi - ig\,\bar{\psi}\gamma_5\psi\,\phi,$$

where ϕ is a real scalar field and ψ is a Dirac fermion. The Lagrangean is invariant under parity transformations defined by

$$\psi(t,\vec{x}) \to \gamma^0\,\psi(t,-\vec{x})$$
$$\phi(x,\vec{x}) \to -\phi(t,-\vec{x})\ ,$$

which implies that ϕ is odd (pseudoscalar). You have computed the counterterms δ_ψ, δ_ϕ, δ_g and δ_λ. They are (up to finite and μ-independent pieces):

$$\begin{aligned}
\delta_\psi &= -\frac{g^2}{32\pi^2}\left(\frac{2}{\epsilon} - \ln\mu^2\right) \\
\delta_\phi &= -\frac{g^2}{8\pi^2}\left(\frac{2}{\epsilon} - \ln\mu^2\right) \\
\delta_g &= \frac{g^3}{16\pi^2}\left(\frac{2}{\epsilon} - \ln\mu^2\right) \\
\delta_\lambda &= \frac{3\lambda^2 - 48g^4}{32\pi^2}\left(\frac{2}{\epsilon} - \ln\mu^2\right)
\end{aligned}$$

Using these, compute the beta functions $\beta(g)$ and $\beta(\lambda)$ to first order in the couplings, assuming λ and g^2 are of the same order.

3. **RG Description of the Ferromagnetic Transition**: (3.5 pts.)
The Landau-Ginzburg description of a ferromagnet corresponds to d dimensional Euclidean ϕ^4 theory, with the action

$$S[\phi] = \int d^d x \left\{ \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2} m^2 \, \phi^2 + \frac{\lambda}{4!} \phi^4 \right\} ,$$

(a) Obtain the leading order shifts for λ and m^2 when rescaling dow to lower energies or larger distance scales. That is, the scale shift $x \to x/b$ plus integrating the high energy modes from $\Lambda/b \to \Lambda$, where Λ is the UV cutoff.

(b) Obtain the renormalization group equation for this shift in length scale, i.e. obtain

$$\frac{d\lambda}{d \ln b} \qquad \text{and} \qquad \frac{dm^2}{d \ln b}$$

to leading order in the ϵ expansion, where $\epsilon = 4 - d$. Show that this results in an IR-stable fixed point for

$$\lambda_* = \frac{16\pi^2}{3} \, \epsilon \qquad m_*^2 = -\frac{\epsilon}{3}$$

What is the behavior in the vicinity of the Gaussian fixed point

$$\lambda_* = 0 \qquad m_*^2 = 0 \text{ ?}$$