Lecture 9

Functional Integral in Quantum Field Theory

Here we will generalize a lot of the results obtained in the path integral formulation of quantum mechanics for quantum field theory. In the case of quantum mechanics, our dynamical variable, the one we integrated over, was $x(t)$, so we were integrating over all possible trajectories. In the case of quantum field theory our dynamical variable will be the field (or fields) $\phi(x)$. So now we have

$$\int \mathcal{D}\phi(x) \ , \tag{9.1}$$

symbolizing the sum over all possible field configurations $\phi(x)$.

Since, as we mentioned in the previous lecture, we will be interested in obtaining the expectation values of operators in the ground state, we will go a more direct route this time. We compute the amplitude to go from one ground state at an initial time t_i to another one at a time t_f. Then, we can use the generating functional formalism in order to obtain the desired correlation functions, i.e. the ground-state-expectation values of time-ordered pruducts of fields.

The starting point is to consider an initial ground state field configuration, symbolized by the state $|0_i\rangle$ at time t_i, and a final ground state configuration at time t_f, $|0_f\rangle$. We want to compute the amplitude

$$\langle 0_f | 0_i \rangle = \langle 0 | e^{-iH(t_f - t_i)} | 0 \rangle \ . \tag{9.2}$$

where now the hamiltonian is that of a real scalar field $\phi(x)$ given by

$$\begin{aligned} H &= \int d^3x \left\{ \frac{1}{2}\hat{\pi}^2 + \frac{1}{2}(\nabla\hat{\phi})^2 + \frac{1}{2}m^2\hat{\phi}^2 \right\} + \ldots \\ &= \int d^3x \left\{ \frac{1}{2}\hat{\pi}^2 + V[\hat{\phi}] \right\} \ , \end{aligned} \tag{9.3}$$

where we used hats to denote operators for the purpose of this derivation. In the first line the dots denote a possible interaction term, and the second line we define the functional $V[\hat{\phi}]$ as the part of the hamiltonian density that does not depend on the conjugate momentum density operator $\hat{\pi}$.

Just as we did for quantum mechanics, we can imagine discretising the time interval $t_f - t_i = n \times \Delta t$ for infinitesimal Δt, and inserting intermediate *field configurations* at intermediate times that are eigenstates of the field operator ϕ, $|\phi_j\rangle$, denoting a field configuration $\phi_j(\mathbf{x})$ at time t_j. So we have

$$\hat{\phi}(\mathbf{x},t)|\phi_j(\mathbf{x})\rangle = \phi_j(\mathbf{x})|\phi_j\rangle \ . \tag{9.4}$$

Each insertion of an itermediate field configuration corresponds to inserting the identitiy as in

$$\int d\phi_j(\mathbf{x})\, |\phi_j\rangle\langle|\phi_j| = 1 \ . \tag{9.5}$$

Strictly speaking, the expression above is already a funcional integral since we are integrating over all possible spatial field configurations $\phi_j(\mathbf{x})$ at the time t_j. Thus, we are actually discretizing spacetime altogether. However, for the prupose of our derivation we will not need the spatial discretization.[1] We then can write

$$\begin{aligned}\langle 0_f|0_i\rangle \ &= \ \int d\phi_1(\mathbf{x})\ldots d\phi_n(\mathbf{x})\langle 0|e^{-iH\Delta t}|\phi_n\rangle\langle\phi_n|e^{-iH\Delta t}|\phi_{n-1}\rangle\langle\phi_{n-1}|\ldots \\ &\quad \langle\phi_{j+1}|e^{-iH\Delta t}|\phi_j\rangle\ldots|\phi_1\rangle\langle\phi_1|e^{-iH\Delta t}|0\rangle \ , \end{aligned} \tag{9.6}$$

where now we can concentrate on computing each of the matrix elements between field configurations separated by the infinitesimal time Δt, analogously to what we did in quantum mechanics. Then we want to compute

$$\langle\phi_{j+1}|e^{-iH\Delta t}|\phi_j\rangle = \int d\pi_j(\mathbf{x})\, \langle\phi_{j+1}|\pi_j\rangle\langle\pi_j|e^{-iH\Delta t}|\phi_j\rangle \ , \tag{9.7}$$

where we have used the insertion of the identity now in the form of a complete set of the eigenstates of the conjugate momentum $|\pi_j\rangle$, which are states of definitive value of the *conjugate momentum density* operator π at time t_j, i.e. we used

$$\int d\pi_j(\mathbf{x})\, |\pi_j\rangle\langle\pi_j| = 1 \ , \tag{9.8}$$

with

$$\hat{\pi}(\mathbf{x},t)|\pi_j\rangle = \pi_j(\mathbf{x})|\pi_j\rangle \ . \tag{9.9}$$

To compute the right hand side of (9.7) we need to remember that the fact that π is the conjugate momentum density associated to the field ϕ means that

$$\langle\pi|\phi\rangle = e^{-i\int d^3x\, \pi(\mathbf{x})\phi(\mathbf{x})} \ , \tag{9.10}$$

[1] The discretization of the spatial coordinates can be seen as a first step in order to already obtain expressions like that in (9.5).

which is the projection of the state $|\phi\rangle$ in conjugate momentum (density) space. Next, to compute the factor $\langle\pi_j|e^{-iH\Delta t}|\phi_j\rangle$ we need to act with the hamiltonian operator H on the states $|\pi_j\rangle$ and $|\phi_j\rangle$. For this, we simply use (9.9) as well as that

$$V[\hat{\phi}]\,|\phi_j\rangle = V(\phi_j(\mathbf{x}))\,|\phi_j\rangle \ . \tag{9.11}$$

The result for (9.7) is

$$\begin{aligned}\langle\phi_{j+1}|e^{-iH\Delta t}|\phi_j\rangle &= \int d\pi_j(\mathbf{x})\, e^{-i\int d^3x\,[\pi_j(\mathbf{x})\phi_j(\mathbf{x})-\pi_j(\mathbf{x})\phi_{j+1}(\mathbf{x})]}\, e^{-i\int d^3x\,\left[\pi_j^2(\mathbf{x})/2+V(\phi_j(\mathbf{x}))\right]\Delta t} \\ &= \int d\pi_j(\mathbf{x})\, e^{-i\int d^3x\,\Delta t\left\{\pi_j^2(\mathbf{x})/2-\pi_j(\mathbf{x})\,\frac{\phi_{j+1}(\mathbf{x})-\phi_j(\mathbf{x})}{\Delta t}+V(\phi_j(\mathbf{x}))\right\}}\ ,\end{aligned} \tag{9.12}$$

where, in the first line, the first exponential just reflects the product $\langle\phi_{j+1}|\pi_j\rangle\langle\pi_j|\phi_j\rangle$, whereas the second exponential corresponds to the action of the hamiltonian on the states $|\phi_j\rangle$ and $|\pi_j\rangle$. The second line in (9.12) can be rewritten to complete the square in π_j giving

$$\begin{aligned}\langle\phi_{j+1}|e^{-iH\Delta t}|\phi_j\rangle &= e^{i\Delta t\int d^3x\left\{\frac{1}{2}\frac{(\phi_{j+1}(\mathbf{x})-\phi_j(\mathbf{x}))^2}{(\Delta t)^2}-V(\phi_j(\mathbf{x}))\right\}}\int d\pi_j(\mathbf{x})\, e^{-i\Delta t\int d^3x\,\frac{1}{2}\left[\pi_j(\mathbf{x})-\frac{(\phi_{j+1}(\mathbf{x})-\phi_j(\mathbf{x}))}{\Delta t}\right]^2} \\ &= N_j\, e^{i\Delta t\int d^3x\,\mathcal{L}[\phi_j,\partial_t\phi_j]}\ .\end{aligned} \tag{9.13}$$

In the second line in (9.13) we performed the gaussian integral in π_j resulting in a constant factor N_j, and we defined the Lagrangian density already anticipating the $\Delta t\to 0$ limit

$$\mathcal{L}[\phi_j,\partial_t\phi_j] = \frac{1}{2}(\partial_t\phi)^2 - V[\phi_j] = \frac{1}{2}(\partial_t\phi)^2 - (\nabla\phi_j)^2 - \frac{1}{2}m^2\phi_j^2 + \dots\ , \tag{9.14}$$

where the dots refer to possible interaction terms. Using (9.13) in (9.6) we then obtain

$$\langle 0_f|0_i\rangle = N\int \mathcal{D}\phi\, e^{iS[\phi]}\ , \tag{9.15}$$

where we used

$$\int \mathcal{D}\phi e^{iS[\phi]} = \lim_{n\to\infty,\Delta t\to 0}\int d\phi_1(\mathbf{x})\dots d\phi_n(\mathbf{x})\prod_{j=1}^{n} e^{i\Delta t\mathcal{L}[\phi_j,\partial_t\phi_j]}\ , \tag{9.16}$$

and the action is defined as usual as

$$S[\phi] = \int d^4x\,\mathcal{L}[\phi,\partial_\mu\phi]\ . \tag{9.17}$$

Since, in general, we will be taking $t_i\to-\infty$ and $t_f\to\infty$, we will drop the time labels to write

$$\boxed{\langle 0|0\rangle = N\int \mathcal{D}\phi\, e^{iS[\phi]}}, \tag{9.18}$$

which says that, up to a normalization factor N, the functional integral over all the possible field configurations is the so-called ground state (or vacuum) persistance. As we will see below, the normalization will not be important in computing observables. The building blocks to do this are the correlation functions we define below.

9.1 Correlation Functions

We define the correlation functions (or n-point functions) of a theory by

$$G^{(n)}(x_1, \dots x_n) \equiv \langle 0|T(\phi(x_1)\dots\phi(x_n))|0\rangle \ . \tag{9.19}$$

For instance, the 2-point function

$$G^{(2)}(x_1, x_2) = \langle 0|T(\phi(x_1)\phi(x_2))|0\rangle \ , \tag{9.20}$$

is the propagator or Green function of the theory. Following the derivation of (9.18) in the previous section it is straightforward to see that[2]

$$G^{(n)}(x_1, \dots x_n) = N \int \mathcal{D}\phi(x)\ \phi(x_1)\dots\phi(x_n)\, e^{iS[(\phi(x),\partial_\mu\phi(x))]} \ . \tag{9.21}$$

So, if we assume that the vacuum persistence is unity, i.e. $\langle 0|0\rangle = 1$, then we obtain

$$G^{(n)}(x_1, \dots x_n) = \frac{\int \mathcal{D}\phi(x)\ \phi(x_1)\dots\phi(x_n)\, e^{iS[(\phi(x),\partial_\mu\phi(x))]}}{\int \mathcal{D}\phi(x)\, e^{iS[(\phi(x),\partial_\mu\phi(x))]}} \ . \tag{9.22}$$

Then, the correlation functions, which are the observables of the theory, are defined in terms of the path integrals. We will use them eventually to compute amplitudes for processes of interest. For now we focus on defining them in a scheme that will allow us to compute them, first in configuration space, and eventually in momentum space. We will make use of the generating functional formalism, which will allow an intuitive way to introduce the concept of perturbation theory in quantum field theory once we introduce interactions.

9.2 Generating Functional in Quantum Field Theory

Before we define the generating functional of a quantum field theory, we will reestablish the definition and some of the properties of functional derivatives in spacetime. Assuming a source that depends on the spacetime position, i.e. $J(x)$, we define the functional derivative as

$$\frac{\delta J(y)}{\delta J(x)} = \delta^{(4)}(x-y) \ , \tag{9.23}$$

[2]For this is enough to notice that for each factor of $\phi(x_j)$ corresponding to an intermediate time t_j, we need the *operator* $\phi(x)$ acting on a state $|\phi_j\rangle$.

which is equivalent to

$$\frac{\delta}{\delta J(x)} \int d^4y\, J(y)\, \phi(y) = \phi(x) \ . \tag{9.24}$$

A useful example is the derivative of the exponential integral. It gives

$$\frac{\delta}{\delta J(x)}\, e^{i\int d^4y\, J(y)\, \phi(y)} = i\phi(x)\, e^{i\int d^4y\, J(y)\, \phi(y)} \ , \tag{9.25}$$

which can be easily proven by expanding the exponential as

$$e^{i\int d^4y\, J(y)\, \phi(y)} = 1+i\int d^4y\, J(y)\, \phi(y)+\cdots+\frac{(i)^n}{n!}\int d^4y_1 J(y_1)\phi(y_1)\cdots\int d^4y_n J(y_n)\phi(y_n)+\ldots \ , \tag{9.26}$$

and using (9.24).

We are now ready to define the generating functional in the presence of an external source $J(x)$ linearly coupled to the field $\phi(x)$. In analogy with quantum mechanics we define

$$Z[J] \equiv N \int \mathcal{D}\phi\, e^{i\int d^4x \{\mathcal{L}[\phi,\partial_\mu\phi]+J(x)\phi(x)\}} \ . \tag{9.27}$$

Then, to obtain the correlation functions we have

$$\boxed{G^{(n)}(x_1,\ldots,x_n) = (-i)^n \frac{1}{Z[0]}\, \frac{\delta^n}{\delta J(x_1)\ldots\delta J(x_n)} Z[J]\bigg|_{J=0}} \tag{9.28}$$

in complete analogy what our result in quantum mechanics. We will work out some simple examples to get a sense of the meaning of these definitions.

9.3 Example: Free Real Scalar Field

Let us consider a free real scalar field. The lagrangian density is

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\, \partial_\mu\phi - \frac{1}{2}m^2\phi^2 \ . \tag{9.29}$$

Integrating by parts in the action we have

$$\int d^4x\,\mathcal{L} = \int d^4x \left(\frac{-1}{2}\right) \phi(x) \left(\partial_\mu\partial^\mu + m^2\right) \phi(x) + \frac{1}{2}\int d^4x\,\partial_\mu(\phi(x)\partial^\mu\phi(x)) \;, \quad (9.30)$$

where the last term is a total derivative and then it can be dropped. The differential operator in the first integral can be defined as

$$O_x \equiv \partial_x^2 + m^2 \;, \quad (9.31)$$

where the subscript denotes derivation with respect to x. In this way, the generating functional in the presence of a linearly coupled source $J(x)$ can be written as

$$Z[J] = N \int \mathcal{D}\phi\; e^{-i\int d^4x\left[\frac{1}{2}\phi(x)\,O_x\,\phi(x) - J(x)\phi(x)\right]} \;. \quad (9.32)$$

Just as for the case of quantum mechanics, we now want to shift the variable $\phi(x)$ in order to separate the source $J(x)$ from the variable of integration in the path integral. For this purpose we define

$$\phi'(x) \equiv \phi(x) - i\int d^4y\, D_F(x-y)\, J(y) \quad (9.33)$$

We will prove in what follows that this will do the job of decoupling the field from the source $J(x)$ as long as the following is satisfied

$$O_x\, D_F(x-y) = -i\delta^{(4)}(x-y) \;. \quad (9.34)$$

That means that the decoupling will take place as long as the function $D_F(x-y)$ is the Green function of the Klein-Gordon operator O_x. Replacing

$$\phi(x) = \phi'(x) + i\int d^4y\, D_F(x-y)\, J(y) \quad (9.35)$$

in (9.32), we have for the exponent

$$\begin{aligned} -i\int d^4x \Big\{ &\frac{1}{2}\phi'(x)\,O_x\,\phi'(x) + \frac{1}{2}\phi'(x)\,J(x) + \frac{i}{2}\int d^4y\, D_F(x-y)\,J(y)\,O_x\,\phi'(x) \quad (9.36) \\ &+\frac{i}{2}\int d^4y J(x)\,D_F(x-y)\,J(y) - J(x)\,\phi'(x) - i\int d^4y J(x)\,D_F(x-y)\,J(y) \Big\} \;. \end{aligned}$$

The third term can be integrated by parts *twice* such that, using (9.34), we have

$$\frac{i}{2}\int d^4y\, D_F(x-y)\, J(y)\, O_x\, \phi'(x) = \frac{1}{2}\phi'(x)\, J(x) + \text{surface terms} \;. \tag{9.37}$$

In this way, we arrive at the generating functional

$$Z[J] = N\int \mathcal{D}\phi\; e^{-i\int d^4x\left\{\frac{1}{2}\phi'(x)O_x\phi'(x) - \frac{i}{2}\int d^4y\, J(x)\, D_F(x-y)\, J(y)\right\}} \;, \tag{9.38}$$

We will assume that the transformation (9.33) leaves the measure of the path integral invariant so that

$$\int \mathcal{D}\phi = \int \mathcal{D}\phi' \;. \tag{9.39}$$

This will usually be the case. Whenever a field transformation does not leave the measure of the field integral invariant we will be in the presence of an anomaly. We will consider that situation in the second part of the course. For now, we will stick to theories where (9.39) holds. Then the generating functional is now

$$Z[J] = N\int \mathcal{D}\phi'\, e^{i\int d^4x\,\mathcal{L}[\phi']}\; e^{-\frac{1}{2}\int d^4x\, d^4y J(x)\, D_F(x-y)\, J(y)} \;, \tag{9.40}$$

which can be rewritten as

$$\boxed{Z[J] = Z[0]\, e^{-\frac{1}{2}\int d^4x\, d^4y\, J(x)\, D_F(x-y)\, J(y)}} \;. \tag{9.41}$$

We see that, just as for the example of the harmonic oscillator in quantum mechanics, the decoupling of the linear term resulted in a genereting functional where all the interesting information comes from the Green function $D_F(x-y)$. As we will see below, all the correlation functions will be determined by it. We are now in a position to use (9.28) to obtain the n-point functions of the scalar theory.

Two-point Function:

We start with the two-point function of the real scalar field theory. By using (9.28) applied on (9.41) we have

$$
\begin{aligned}
G^{(2)}(x_1,x_2) &= \frac{(-i)^2}{Z[0]}\,\frac{\delta^2}{\delta J(x_1)\delta J(x_2)} Z[J]\Big|_{J=0} && (9.42)\\
&= (-1)\frac{\delta}{\delta J(x_1)}\left\{-\frac{1}{2}\int d^4y D_F(x_2-y)\,J(y) - \frac{1}{2}\int d^4x J(x)\,D_F(x-x_2)\right\}\Big|_{J=0}\\
&= (-1)\left\{-\frac{1}{2}D_F(x_2-x_1) - \frac{1}{2}D_F(x_1-x_2)\right\} = D_F(x_1-x_2)\ .
\end{aligned}
$$

So we arrive at

$$
G^{(2)}(x_1,x_2) = D_F(x_1-x_2) = \langle 0|T(\phi(x_1)\phi(x_2))|0\rangle\ . \tag{9.43}
$$

Of course, this confirms that the two-point function is the Feynman propagator. Incidentally, all we asked for the function to do is for it to be a Green function of O_x, i.e. to satisfy (9.34). It is straightforward to prove that $D_F(x-y)$ is given by

$$
D_F(x-y) = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{i}{k^2-m^2}\ . \tag{9.44}
$$

<u>Four-point Function</u>:

We can follow the same procedure to compute the four-point function

$$
G^{(4)}(x_1,x_2,x_3,x_4) = \langle 0|T(\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4))|0\rangle\ . \tag{9.45}
$$

The result is

$$
\begin{aligned}
G^{(4)}(x_1,x_2,x_3,x_4) &= D_F(x_1-x_2)\,D_F(x_3-x_4) + D_F(x_1-x_3)\,D_F(x_2-x_4)\\
&\quad + D_F(x_1-x_4)\,D_F(x_2-x_3)\ . && (9.46)
\end{aligned}
$$

This can be represented diagrammatically, as we can seen in Figure 9.1. We can see that the four-point function of the free theory is not very exciting. We can only have free propagation joining the two pairs of points available in each possible pairing. We can also see that the n-point function is zero for n odd. This is clear from the fact that when the external sources are turned off ($J \to 0$) all odd n correlation functions vanish. Then we can generalize this for $2n$-point functions defined as

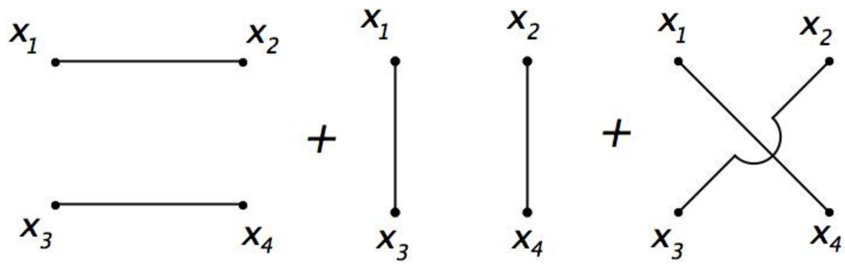

Figure 9.1: The three Feynman diagrams contributing to the four-point function of the free real scalar field.

$$G^{(2n)}(x_1, \ldots, x_{2n}) = \sum_{\text{all pairings}} D_F(x_{i_1} - x_{i_2}) \ldots D_F(x_{2n-1} - x_{2n}) \ . \tag{9.47}$$

So the $2n$-point function is a sum over the products of n propagators paired in all possible ways. This statement is called Wick Theorem.

9.4 Integrating Quadratic Forms

All throughout the course we will need to integrate quadratic forms. We will reduce them to Gaussian integrals, which we know how to perform. Let us start from generating functional for $J = 0$ in the free real scalar theory from the previous section.

$$Z[0] = N \int \mathcal{D}\phi \, e^{-i \int d^4x \, \phi(x) \, O_x \, \phi(x)} \ , \tag{9.48}$$

where absorb the factor of 1/2 in the definition of the operator. That is, we have

$$O_x = \frac{1}{2} \left(\partial^2 + m^2 \right) \ . \tag{9.49}$$

The point is to be able to write (9.48) above as a quadratic form in the fields.

In general, if the action can be written as

$$S = \int d^4x \, \phi(x) \, \mathcal{O} \, \phi(x) \ , \tag{9.50}$$

where $\mathcal{O}$ is a differential operator that does not depend on ϕ (e.g. it could depend on other fields), then the functional integral can be performed.

To start, we expand the field $\phi(x)$ in terms of eigenfunctions of the operator $\mathcal{O}$

$$\phi(x) = \sum_n a_n \varphi_n(x) \, , \tag{9.51}$$

such that

$$\mathcal{O} \varphi_n(x) = \lambda_n \varphi_n(x) \, , \tag{9.52}$$

with the $\{\lambda_n\}$ the eigenvalues. In addition, the eigenfunctions satisfy the orthonormality conditions

$$\int d^4x \, \varphi_n(x) \, \varphi_m(x) = \delta_{nm} \, . \tag{9.53}$$

We will typically try to compute integrals of the sort

$$I = \int \mathcal{D}\phi \, e^{-i \int d^4x \, \phi(x) \, \mathcal{O} \, \phi(x)} \, . \tag{9.54}$$

Making use of (9.51) we have

$$I = N \left(\prod_n \int_{-\infty}^{+\infty} da_n \right) e^{-i \int d^4x \, \sum_{k=1}^{\infty} a_k \, \varphi_k(x) \times \sum_{\ell=1}^{\infty} \lambda_\ell \, a_\ell \, \varphi_\ell(x)} \, , \tag{9.55}$$

where the integration over all possible field configurations is now the (infinite) product of the integrals over all possible values of the expansion coefficients a_n. The normalization N is irrelevant, as usual. If we now use the orthonormality condition (9.53) we obtain

$$I = N \prod_n \int_{-\infty}^{+\infty} da_n \, e^{-i\lambda_n \, a_n^2} = N \prod_n \sqrt{\frac{\pi}{i\lambda_n}} \tag{9.56}$$

This can be further simplified as

$$I = N' \prod \frac{1}{\sqrt{\lambda_n}} = N' \sqrt{\frac{1}{\prod_n \lambda_n}} \, , \tag{9.57}$$

where we have absorbed some constants in N', a new normalization. Noticing that

$$\prod_{n=1}^{\infty} \lambda_n = \det \mathcal{O} \ , \tag{9.58}$$

i.e. the determinant of the differential operator $\mathcal{O}$, we arrive at

$$\boxed{I = N' \left(\det \mathcal{O}\right)^{-1/2}} \ . \tag{9.59}$$

The result in (9.59) is generic for any functional integral that is quadratic in *bosonic* fields. The actual value of the determinant is not important in this case, but in general it would be (e.g. in the presence of interactions). For the purposes we will use this result it is useful to notice that we can write

$$\det \mathcal{O} = e^{\mathrm{Tr} \ln \mathcal{O}} \ . \tag{9.60}$$

To convince ourselves that this is correct, let us write

$$e^{\mathrm{Tr} \ln \mathcal{O}} = e^{\sum_n \ln \lambda_n} = \prod_n e^{\ln \lambda_n} = \prod_n \lambda_n = \det \mathcal{O} \ . \tag{9.61}$$

The trace includes all indices and also all the possible values of the spacetime points, i.e.

$$\mathrm{Tr} \ln \mathcal{O} = \int \mathrm{d}^4\mathrm{x} \, \langle \mathrm{x} | \ln \mathcal{O} | \mathrm{x} \rangle \tag{9.62}$$

Exponentiation of the determinant will be of importance since we will be able to include it effectively as a new term in the action. Performing the functional integral over a field is referred to as having *integrated out* the field in question. As we will see later, this procedure is very useful, particularly when the fields become too heavy in comparison with the typical energy scale of the problem.

Additional suggested readings

- *Quantum Field Theory* , by M. Srednicki, Chapter 8.
- *Quantum Field Theory in a Nutshell*, by A. Zee. Chapter 1.7, first few sections on free field theory.