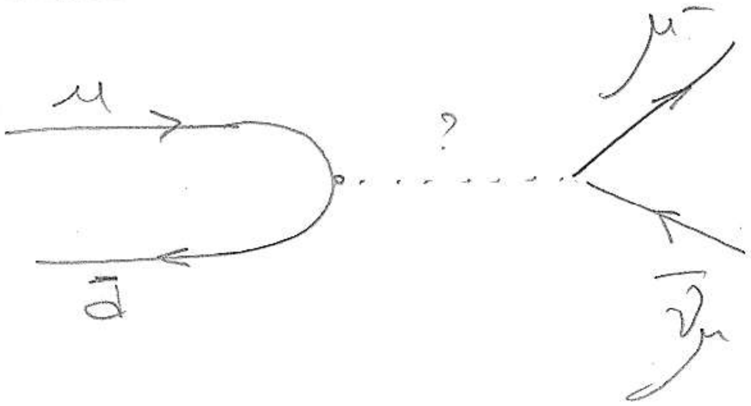
Interações Fracas II

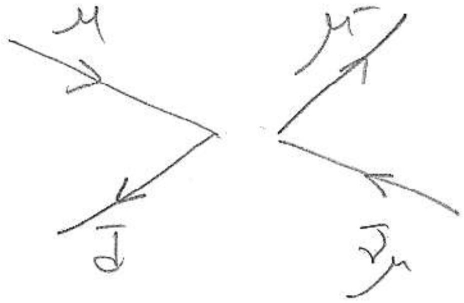
(A19) ①

Decaimento do π

$$\pi^- \to \mu^- \bar{\nu}_\mu \; ; \; e^- \bar{\nu}_e$$

π^- é $u\bar{d}$!





$$\pi^-(q) \to \mu^-(p)\, \bar{\nu}(k)$$

$$q_\mu = p_\mu + k_\mu$$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \, \bar{d}\, \gamma_\mu (1-\gamma_5)\, u \; \bar{\mu}\, \gamma^\mu (1-\gamma_5)\, \nu_\mu$$

↳ Processo em nível "partônico" (de quarks)!

O verdadeiro processo involve π^-!

②

$\Rightarrow$ Escrever a corrente carregada de quarks para como

$$j^{\pi}_{\mu} = \bar{d}\gamma_\mu(1-\gamma_5)u \quad \text{tal que}$$

$$\langle \pi^-(q) | j^{\pi^-}_\mu | 0 \rangle$$

é o elemento de matriz do operador $j^{\pi^-}_\mu$ entre $\langle \pi^-(p)|$ e $|0\rangle$!

$$\pi^- \longrightarrow ? \longrightarrow \bar{\nu}_\mu \mu^-$$

Parametrizar: Não sabemos calcular o elemento de matriz "hadrônico". Mas ele deve ter a forma:

$$\langle \pi^-(q_\mu) | j^{\pi}_\mu | 0 \rangle = q_\mu * f(q^2)$$

q^2 único escalar de Lorentz formado com q_μ. MAS $q^2 = m^2_\pi$!

$\Rightarrow$ Chamamos de

$$f(q^2 = m^2_\pi) \equiv f_\pi$$

constante de decaimento do píon

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$$\mathcal{M} = \frac{G_F}{\sqrt{2}} f_\pi \left(p^\mu + k^\mu\right) \bar{\mu} \gamma_\mu (1-\gamma^5) \nu_\mu$$

Se $m_\nu \simeq 0 \Rightarrow$

$$k^\mu \gamma_\mu \nu_\mu = \not{k} \nu_\mu = -m_\nu \nu_\mu = 0$$

$$\bar{\mu}\, p^\alpha \gamma_\alpha = \bar{\mu} \not{p} = m_\mu \bar{\mu} \quad \text{vindo}$$

de
$$(\not{p} - m_\mu)\, \mu(\vec{p}) = 0$$
$$\bar{\mu}(p)(\not{p} - m_\mu) = 0$$

$$\Rightarrow \boxed{\mathcal{M} = \frac{G_F}{\sqrt{2}} f_\pi m_\mu \bar{\mu} (1-\gamma^5) \nu_\mu}$$

$$|\mathcal{M}|^2 = \frac{G_F^2}{2} f_\pi^2 m_\mu^2 \,\mathrm{Tr}\left[\not{p} (1-\gamma^5) \not{k} (1+\gamma^5)\right] \quad \not{p} + m_\mu$$

não do contrib.

$$= \ldots \left\{ \mathrm{Tr}[\not{p}\not{k}] - \mathrm{Tr}[\not{p}\gamma^5 \not{k} \gamma^5] - \underset{0}{\underbrace{\mathrm{Tr}[\not{p}\gamma^5 \not{k}]}} + \underset{0}{\underbrace{\mathrm{Tr}[\not{p}\not{k}\gamma^5]}} \right.$$

[illegible]

$$|\mathcal{M}|^2 \sim \bar{\mu}_\alpha (1-\gamma^5)_{\alpha\beta} \nu_\beta \left(\mu^\dagger \gamma^0 (1-\gamma^5)\nu\right)^\dagger$$

$$\sim \bar{\mu}_\alpha (1-\gamma^5)_{\alpha\beta} \nu_\beta \, \nu^\dagger (1-\gamma^5) \gamma^{0\dagger} \mu$$

$$\sim \bar{\mu}_\alpha (1-\gamma^5)_{\alpha\beta} \nu_\beta \, \nu^\dagger_\gamma \gamma^0 (1+\gamma^5) \mu = \bar{\mu}_\alpha (1-\gamma^5)_{\alpha\beta} \underbrace{\nu_\beta \bar{\nu}_\gamma} (1+\gamma^5)_{\gamma\delta} \mu_\delta$$

$$\bar{\mu}_\alpha (1-\gamma^5)_{\alpha\beta} \not{k}_{\beta\gamma} (1+\gamma^5)_{\gamma\delta} \mu_\delta$$

$$\sim \left(\not{p} + m_\mu\right)_{\delta\alpha} (1-\gamma^5)_{\alpha\beta} \not{k}_{\beta\gamma} (1+\gamma^5)_{\gamma\delta}$$

$$\Rightarrow |\mathcal{M}|^2 = \frac{G_F^2}{2} f_\pi^2 m_\mu^2 \, \mathrm{Tr}\left[\not{p}(1-\gamma^5)\not{k}(1+\gamma^5)\right]$$

$$= \frac{G_F^2}{2} f_\pi^2 m_\mu^2 \; 2\, \mathrm{Tr}[\not{p}\not{k}] \, .$$

$$\boxed{|\mathcal{M}|^2 = 4 G_F^2 f_\pi^2 m_\mu^2 \; p\cdot k} = \overline{|\mathcal{M}|^2}$$

$$\Downarrow !$$

$$\Gamma = \frac{1}{2E_\pi} \overline{|\mathcal{M}|}^2 \frac{d^3 p_\mu}{(2\pi)^3 2E_\mu} \frac{d^3 p_\nu}{(2\pi)^3 2E_\nu} (2\pi)^4 \delta^{(4)}(p_A - p_\mu - p_\nu)$$ 5

Aqui E_π pode ser no sistema em repouso

$$E_\pi = m_\pi$$

$$\Gamma = \frac{1}{2 \cdot (2\pi)^2 m_\pi} \overline{|\mathcal{M}|}^2 \int \frac{d^3 p_\mu}{E_\mu E_\nu} \delta(\overset{m_\pi}{E_\pi} - E_\nu - E_\mu)$$

$$\Gamma = \frac{1}{8\pi^2 m_\pi} \overline{|\mathcal{M}|}^2 \frac{p_\mu^2 \, dp_\mu}{E_\mu E_\nu} \overbrace{d\Omega_\mu}^{4\pi} \delta(m_\pi - E_\nu - E_\mu)$$

Para fazer a integral:

$$\delta(m_\pi - E_\nu - E_\mu) = \delta\left(m_\pi - \sqrt{p_\mu^2 + m_\mu^2} - \sqrt{p_\mu^2 + m_\nu^2}\right)$$

$$\Rightarrow \Gamma = \frac{4\pi}{8\pi^2 m_\pi} \overline{|\mathcal{M}|}^2 \frac{p_\mu^2}{E_\mu E_\nu} \frac{1}{\left(\frac{p_\mu}{E_\mu} + \frac{p_\mu}{E_\nu}\right)}$$

$$\Gamma = \frac{1}{2\pi m_\pi} \overline{|\mathcal{M}|}^2 \frac{p_\mu}{\underbrace{[E_\mu + E_\nu]}_{= m_\pi}}$$

$$\Gamma = \frac{1}{2\pi m_\pi^2} p_\mu \overline{|\mathcal{M}|^2}$$

onde p_μ: $\quad m_\pi = E_\nu + E_\mu$

$$m_\pi = p_\mu + \sqrt{p_\mu^2 + m_\mu^2}$$

$$m_\pi - p_\mu = \sqrt{p_\mu^2 + m_\mu^2}$$

$$m_\pi^2 + \cancel{p_\mu^2} - 2 m_\pi p_\mu = \cancel{p_\mu^2} + m_\mu^2$$

$$\Rightarrow p_\mu = \frac{m_\pi^2 - m_\mu^2}{2 m_\pi}$$

$$\Rightarrow \boxed{\Gamma = \frac{(m_\pi^2 - m_\mu^2)}{4\pi m_\pi^3} \overline{|\mathcal{M}|^2}}$$

$$\overline{|\mathcal{M}|^2} = 4 G_F^2 f_\pi^2 m_\mu^2 \underline{p \cdot k} =$$

$$p \cdot k = E_\mu E_\nu - \vec{p} \cdot \vec{k} = E_\mu \overset{p_\mu}{\overset{\parallel}{E_\nu}} + p_\mu^2$$

$$=$$

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$$P \cdot K = E_\mu P_\mu + P_\mu^2$$

$$E_\mu = \sqrt{P_\mu^2 + m_\mu^2}$$

$$E_\mu^2 = \frac{(m_\pi^2 - m_\mu^2)^2}{4 m_\pi^2} + \frac{4 m_\pi^2 m_\mu^2}{4 m_\pi^2}$$

$$E_\mu^2 = \frac{(m_\pi^2 + m_\mu^2)^2}{4 m_\pi^2} \Rightarrow E_\mu = \frac{(m_\pi^2 + m_\mu^2)}{2 m_\pi}$$

$$\Rightarrow P_\alpha K^\alpha = \frac{(m_\pi^2 + m_\mu^2)}{2 m_\pi} \frac{(m_\pi^2 - m_\mu^2)}{2 m_\pi} + \frac{(m_\pi^2 - m_\mu^2)^2}{4 m_\pi^2}$$

$$= \frac{(m_\pi^2 - m_\mu^2)}{4 m_\pi^2} \left[m_\pi^2 + m_\mu^2 + m_\pi^2 - m_\mu^2 \right]$$

$$\boxed{P_\alpha K^\alpha = \frac{(m_\pi^2 - m_\mu^2)}{2}}$$

$$\Gamma = \frac{(m_\pi^2 - m_\mu^2)^2}{8\pi\, m_\pi^3}\, 4\, G_F^2 f_\pi^2\, m_\mu^2$$

$$\boxed{\Gamma = \frac{G_F^2}{2\pi} f_\pi^2\, m_\mu^2\, m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2}$$

$f_\pi = ?$ Medido experimentalmente de aqui!

$\sim$ $f_\pi \simeq 132$ MeV! $\simeq m_\pi$!

$G_F = 1.166\ 10^{-5}\ \mathrm{GeV}^{-2}$

$\Rightarrow \Gamma = 5.28\ 10^{-16} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$ GeV

$\Gamma = 1.27\ 10^{-16}$ GeV

$$\tau = \frac{\hbar}{\Gamma} = \frac{6.582\ 10^{-25}\ \mathrm{GeV\ s}}{1.27\ 10^{-16}\ \mathrm{GeV}}$$

$\boxed{\tau_\pi \simeq 5 \times 10^{-9}\,\mathrm{s}}$ A verdade $\sim 2.8\ 10^{-8}$ s! ??

Suppressão por helicidade:

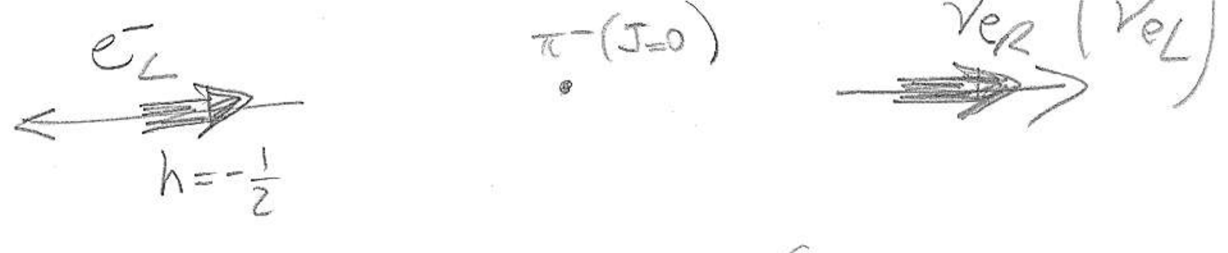
$$\frac{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \to e^- \bar{\nu}_e)} = \frac{m_\mu^2}{m_e^2} \frac{(m_\pi^2 - m_\mu^2)^2}{(m_\pi^2 - m_e^2)^2}$$

$\Rightarrow \Gamma(\pi^- \to \mu^- \bar{\nu}_\mu) \simeq 10^4\, \Gamma(\pi^- \to e^- \bar{\nu}_e)$!

$\Rightarrow$ o π^- decai em μ^-! quase sempre!

Por que!? No limite $m_e \to 0$

π^- não decai!



Se ambos são de mão-esquerda ($\Rightarrow \bar{\nu}_e$ é de mão-direita)

$\Rightarrow$ Não pode decair por cons. do momento!

$J^z_\pi = 0 \qquad J^z_{final} = 1$! ??

Precisa acontecer com ν!

MAS as interações só se acoplam com $\bar{e}_L$!

⇒ Como fazer para $\bar{e}_L \to \bar{e}_R$?

Termo de MASSA!

$$\mathcal{L} = \bar{e}_L i\not{\partial} e_L + \bar{e}_R i\not{\partial} e_R - m_e(\bar{e}_L e_R + \bar{e}_R e_L)$$

!

Por isso que no limite m_e (ou m_μ) ⇒ 0

Não temos decaimento do π!

Esta característica de π^- é inerente às interações fracas!