Teorias de Gauge Não-Abelianas : A16

Assumimos que os campos de matéria (eg férmions) se transformam como

$$\psi \longrightarrow e^{i\alpha^a(x)\,t^a}\,\psi = U(x)\,\psi$$

onde . $\alpha^a(x)$ são funções arbitrárias de x.

. t^a são os geradores do grupo

se é $SU(N)$ são:

* Matrizes de $N\times N$ Unitárias $U(x)$

$\Rightarrow$ t^a são $N\times N$ de traço nulo

$$U^\dagger = U^{-1} \Rightarrow \alpha^{a*}\,t^{a\dagger} = \alpha^a\,t^a$$

$\Rightarrow$ * se t^a são hermitianos $(t^{a\dagger} = t^a)$

$\Rightarrow$ <u>$\alpha^a(x)$ são reais</u>

* $\exists$ $a = 1, \ldots N^2-1$ dessas matrizes

Geradores satisfazem regras de comutação

$$[t^a, t^b] = i f^{abc} t^c$$ (índices somados)

↳ totalmente anti-simétricos

Exemplo: $SU(2)$ $a = 1, 2, 3 \Rightarrow$ pauli matrices

$$[\sigma^a, \sigma^b] = i2\epsilon^{abc}\sigma^c$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = i\sigma^3$$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = -i\sigma^3$$

$$\sigma^1\sigma^2 - \sigma^2\sigma^1 = 2i\sigma^3$$

$$\Rightarrow \boxed{\left[\frac{\sigma^a}{2}, \frac{\sigma^b}{2}\right] = i\epsilon^{abc}\frac{\sigma^c}{2}}$$

$\Rightarrow$ os geradores de $SU(2)$ são

$$\boxed{t^a = \frac{\sigma^a}{2}} \quad !$$

Voltando a $SU(N)$, consideremos uma transf. de gauge infinitesimal: 3

$$\psi \longrightarrow e^{i\alpha^a t^a}\psi \simeq (1 + i\alpha^a t^a)\psi \quad (\alpha^a \ll 1)$$

$$\Rightarrow \mathcal{L} = \bar{\psi} i\gamma^\mu D_\mu \psi - m\bar{\psi}\psi$$

. $\bar{\psi}\psi$ é invariante de Gauge ✓

. Se $\psi \longrightarrow (1 + i\alpha^a t^a)\psi$

$$\Rightarrow \bar{\psi} i\gamma^\mu \partial_\mu \psi \longrightarrow e^{-i\alpha^a t^a}\bar{\psi} i\gamma^\mu \partial_\mu (e^{i\alpha^a t^a}\psi)$$

$$= \bar{\psi} i\gamma^\mu \partial_\mu \psi + (i)^2 \bar{\psi}\gamma^\mu \partial_\mu (\alpha^a t^a)\psi$$

$$= \bar{\psi} i\gamma^\mu \partial_\mu \psi - \bar{\psi}\gamma^\mu t^a (\partial_\mu \alpha^a_{(x)})\psi$$

$\Rightarrow$ Q: que deve ser D_μ?

Se $D_\mu = \partial_\mu - i g\, t^a A_\mu^a$

Como deve se transformar A_μ^a?

A Lagrangiana é

$$\mathcal{L} = \bar{\psi} i \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi + g \bar{\psi} \gamma^\mu t^a \psi A_\mu^a$$

Se a gente usa

$$A_\mu^a \longrightarrow A_\mu^a + \frac{1}{g} \partial_\mu \alpha^a(x)$$

isto cancela o termo que vem de $\partial_\mu \psi \to \partial_\mu \psi'$.
Mas agora temos, vindo do último termo:

$$\bar{\psi} \gamma^\mu t^a \psi \longrightarrow \bar{\psi}(1 - i\alpha^b t^b) \gamma^\mu t^a (1 + i\alpha^b t^b) \psi$$

$$= \bar{\psi} \gamma^\mu t^a \psi + i \bar{\psi} \gamma^\mu t^a t^b \psi \alpha^b - i \bar{\psi} \gamma^\mu t^b t^a \psi \alpha^b + \cdots$$

$$= \bar{\psi} \gamma^\mu t^a \psi + i\alpha^b \bar{\psi} \gamma^\mu (t^a t^b - t^b t^a) \psi + \cdots$$

$$= \bar{\psi} \gamma^\mu t^a + i\alpha^b \bar{\psi} \gamma^\mu [t^a, t^b] \psi$$

$$\Rightarrow \bar{\psi}\gamma^\mu t^a \psi \rightarrow \bar{\psi}\gamma^\mu t^a \psi + i\alpha^b i f^{abc}\bar{\psi}\gamma^\mu t^c \psi$$

$$\rightarrow \bar{\psi}\gamma^\mu t^a \psi - \alpha^b f^{abc}\bar{\psi}\gamma^\mu t^c \psi$$

$\Rightarrow$ Precisamos agregar um termo na transformação dos A_μ^a:

$$\boxed{A_\mu^a \rightarrow A_\mu^a + \frac{1}{g}\partial_\mu \alpha^a + f^{abc} A_\mu^b \alpha^c}$$

E o termo cinético dos campos de Gauge

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$$ é invariante de gauge? Não!

$$G^a_{\mu\nu} \to \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \partial_\mu (f^{abc} \alpha^c A^b_\nu)$$
$$- \partial_\nu (f^{abc} \alpha^c A^b_\mu)$$
$$= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} (\partial_\mu \alpha^c) A^b_\nu - f^{abc} (\partial_\nu \alpha^c) A^b_\mu$$
$$+ f^{abc} \alpha^c \partial_\mu A^b_\nu - f^{abc} \alpha^c \partial_\nu A^b_\mu$$

Se agregamos o termo

$$+ g f^{abc} A^b_\mu A^c_\nu$$

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$$\rightarrow + g f^{abe} \left(A_\mu^b + \frac{1}{g} \partial_\mu \alpha^b + f^{bcd} \alpha^c A_\mu^d \right)$$

$$\times \left(A_\nu^e + \frac{1}{g} \partial_\nu \alpha^e + f^{efg} \alpha^f A_\nu^g \right)$$

$$= + g f^{abe} A_\mu^b A_\nu^e + f^{abe} \partial_\nu \alpha^e A_\mu^b + g f^{abe} \underline{f^{efg} \alpha^f A_\mu^b A_\nu^g}$$

$$+ f^{abe} \partial_\mu \alpha^b A_\nu^e + \frac{f^{abe}}{g} \partial_\mu \alpha^b \partial_\nu \alpha^e \rightarrow 0$$

$$+ \underline{f^{abe} f^{efg} \partial_\mu \alpha^b \alpha^f A_\nu^g}$$

$$+ g f^{abe} f^{bcd} \alpha^c A_\mu^d A_\nu^e + \underline{f^{abe} \alpha^c A_\mu^d \partial_\nu \alpha^e} f^{bcd}$$

$$+ g f^{abe} f^{bcd} f^{efg} \alpha^c \alpha^f A_\mu^d A_\nu^g$$

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$$\Rightarrow \boxed{G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu}$$

é invariante sob transformações

$$A^a_\mu \longrightarrow A^a_\mu + \frac{1}{g}\partial_\mu \alpha^a + f^{abc} A^c_\mu$$

$\Rightarrow$ termo "cinético" dos Campos de Gauge é

$$\boxed{\mathcal{L}_{Gauge} = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}}$$ (somado em $a = 1, \ldots N^2-1$)

$\Rightarrow$ Termo "cinético" dos A^a_μ $\Rightarrow$ interações entre os bósons de Gauge!

Outro jeito

$$\left.\begin{array}{l} D_\mu \psi \longrightarrow U (D_\mu \psi) \\ \psi \to U \psi \\ \bar{\psi} \to \bar{\psi} U^\dagger \end{array}\right] \Rightarrow i\bar{\psi}\not{D}\psi \text{ invariante}$$

$$D_\mu D_\nu \psi \to D_\mu (U D_\nu \psi) \to U (D_\mu D_\nu \psi)$$

$$[D_\mu D_\nu - D_\nu D_\mu]\psi$$

$$\downarrow$$

$$U \underbrace{[D_\mu D_\nu - D_\nu D_\mu]}_{= -ig F^a_{\mu\nu} t^a} \psi$$

$$\boxed{F_{\mu\nu} \to U F_{\mu\nu} U^\dagger}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$F^a_{\mu\nu} t^a \longrightarrow U F^a_{\mu\nu} t^a U^\dagger$$

$$\Rightarrow \mathrm{Tr}\left[F^a_{\mu\nu} t^a F^{b\,\mu\nu} t^b\right] = \frac{1}{2} \left(F^a_{\mu\nu} F^{a\mu\nu}\right)$$

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$$\mathcal{L}_{GAUGE} = -\frac{1}{4}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c\right)$$

$$\times \left(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + g f^{ade} A^{d\nu} A^{e\mu}\right)$$

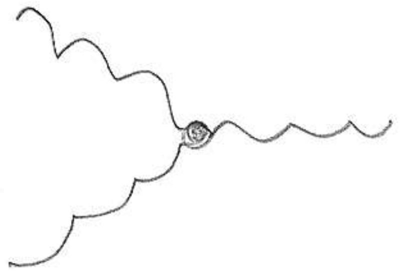
$$= -\frac{1}{4}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right)\left(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu}\right) \Big] \text{ cinético mesmo (2 campos)}$$

$$-\frac{1}{4}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right) g f^{ade} A^{d\nu} A^{e\mu}$$

$$-\frac{1}{4}\left(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu}\right) g f^{abc} A_\mu^b A_\nu^c \Bigg\} \text{ 3 campos} \rightarrow \text{interação}$$

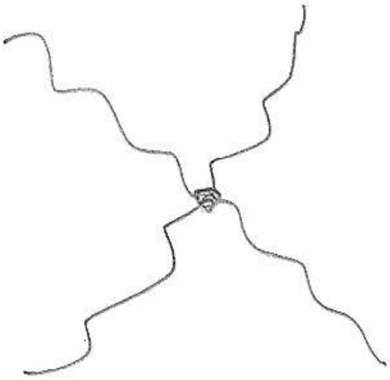
$$-\frac{1}{4} g^2 f^{abc} f^{ade} A_\mu^b A_\nu^c A^{d\nu} A^{e\mu} \Big] \text{ 4 campos}$$

$\Rightarrow$



3 campos de Gauge

$\sim g$



4 campos de Gauge

$\sim g^2$

Campos de Gauge auto-interagentes é uma característica de todas as teorias de Gauge <u>Não-Abelianas</u>.

QCD ;

Teoria de Gauge Não-Abeliana $SU(3)_{cor}$

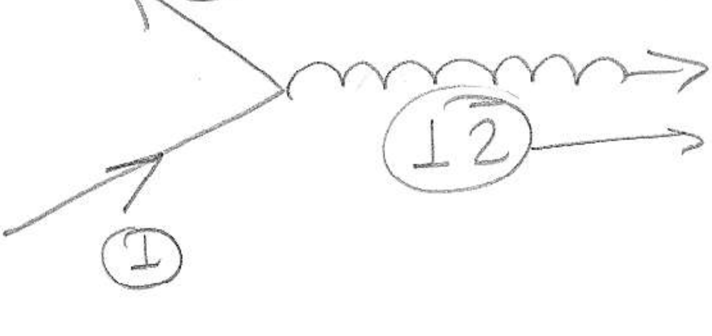
q_α $\alpha = 1, 2, 3$ 3 "Cores"

$\Rightarrow$ Quarks tem um número quântico de "cor" tal que as ψ_{total} dos bárions são anti-simétricas sob $i \leftrightarrow j$.

"Espinores" de cor podem ser representados por ex.

$$1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \; ; \quad 2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \; ; \quad 3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Gluons. tem "cor" mais em uma representação diferente



cor do gluon é "composto"

Quantos glúons podemos ter?

9 possíveis combinações

$$G_1 = \left(1\bar{2} + 2\bar{1}\right)/\sqrt{2}$$

$$G_2 = \frac{-i}{\sqrt{2}}\left(1\bar{2} - 2\bar{1}\right)$$

$$G_3 = \frac{\left(1\bar{1} - 2\bar{2}\right)}{\sqrt{2}}$$

$$G_4 = \frac{\left(1\bar{3} + 3\bar{1}\right)}{\sqrt{2}}$$

$$G_5 = \frac{-i}{\sqrt{2}}\left(1\bar{3} - 3\bar{1}\right)$$

$$G_6 = \frac{\left(2\bar{3} + 3\bar{2}\right)}{\sqrt{2}}$$

$$G_7 = \frac{-i}{\sqrt{2}}\left(2\bar{3} - 3\bar{2}\right)$$

$$G_8 = \frac{\left(1\bar{1} + 2\bar{2} - 2\times 3\bar{3}\right)}{\sqrt{6}}$$

$$\boxed{G_9 = \frac{1\bar{1} + 2\bar{2} + 3\bar{3}}{\sqrt{3}}}$$

→ não tem cor total (= 0!)

⇒ singleto de cor.

Mas! Se postulamos (observamos) que 13

- todas as partículas "observadas" (hadrons) tem cor = 0
- Não $\exists$ bósons de gauge de cor = 0 e $M=0$ fora o fóton.

$\Rightarrow$ <u>G_9 não pode existir!</u>

$\Rightarrow$ Só 8 glúons

$\Rightarrow$ <u>teoria de gauge é SU(3) e não U(3)!</u>

Geradores de SU(3)

$$[t^a, t^b] = i f^{abc} t^c \qquad q \to e^{i\alpha^a(x) t^a} q$$

Matrices de 3x3, traço nulo, hermitianas /

$$q \to U(x)\, q = e^{i\alpha^a t^a} q$$

Matrices de Gell-Mann: Generalizar Pauli

$$\left[\frac{\lambda^a}{2} ; \frac{\lambda^b}{2}\right] = i f^{abc} \frac{\lambda^c}{2}$$

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} ; \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} ; \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} ; \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

Constantes de estrutura de SU(3)

$$f^{abc} = -f^{bac} = f^{cab}$$

As que não são zero são:

$$\begin{cases} f^{123} = 1 \\ f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \frac{1}{2} \\ f^{458} = f^{678} = \frac{\sqrt{3}}{2} \end{cases}$$